Chesapeake Bay Analysis using Time and Spatial Generalized Eigenfunctions

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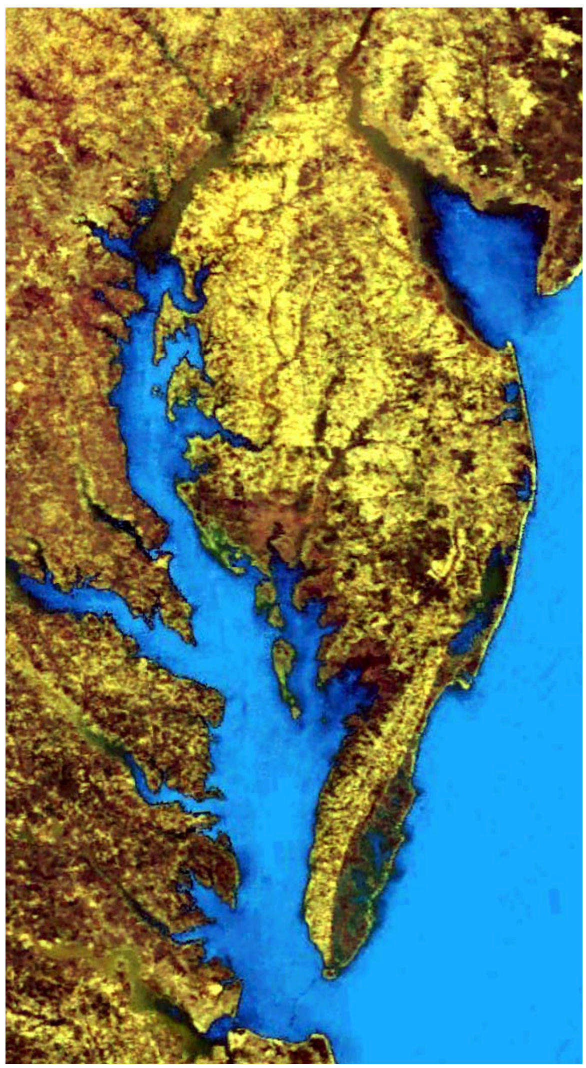

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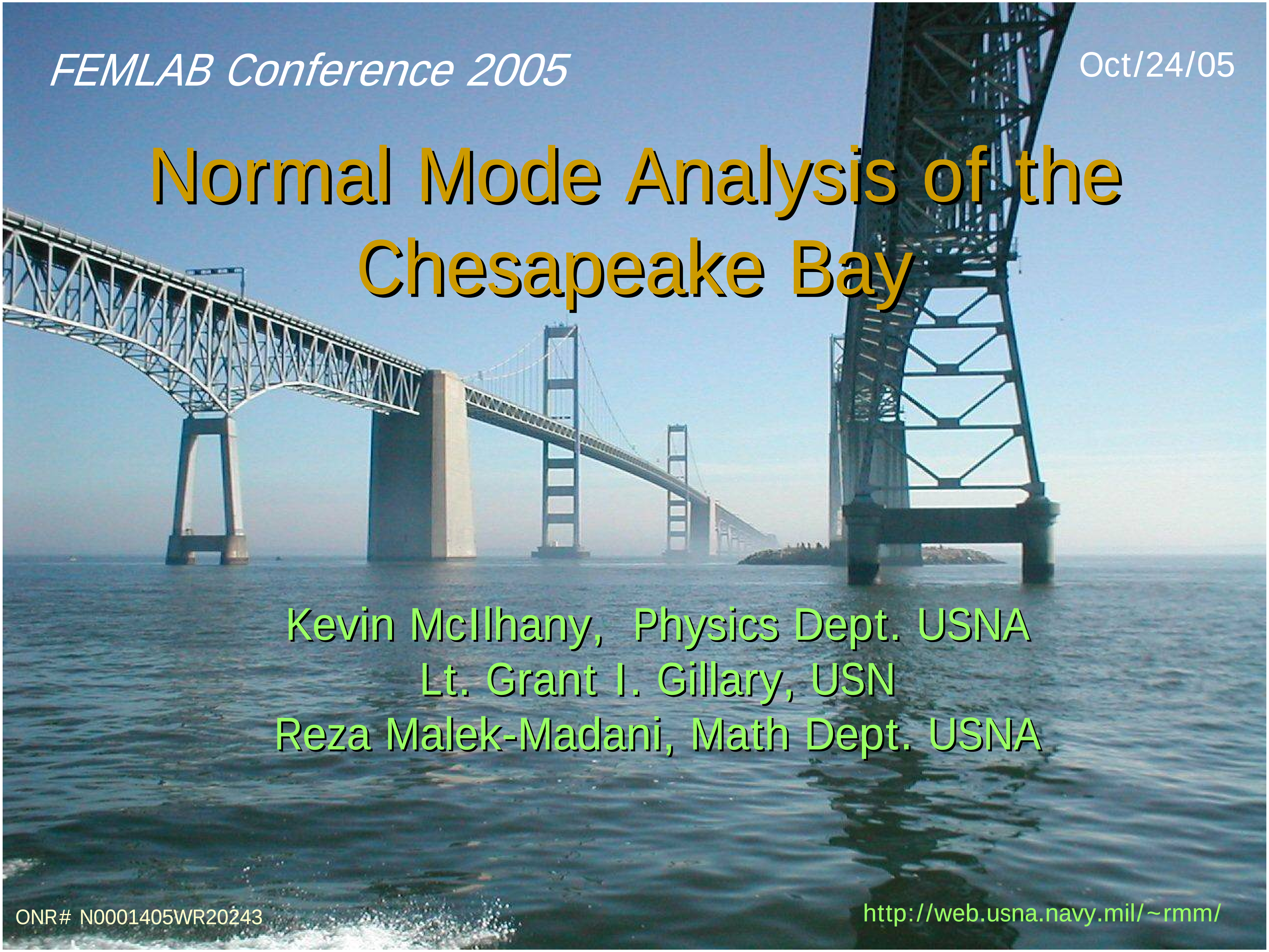

FEMLAB Conference 2005

Oct/24/05

Normal Mode Analysis of the Chesapeake Bay

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ONR# N0001405WR20243

http://web.usna.navy.mil/~rmm/

COMSOL Conference 2006

Oct/23/06

Time Series Analysis of the Chesapeake Bay

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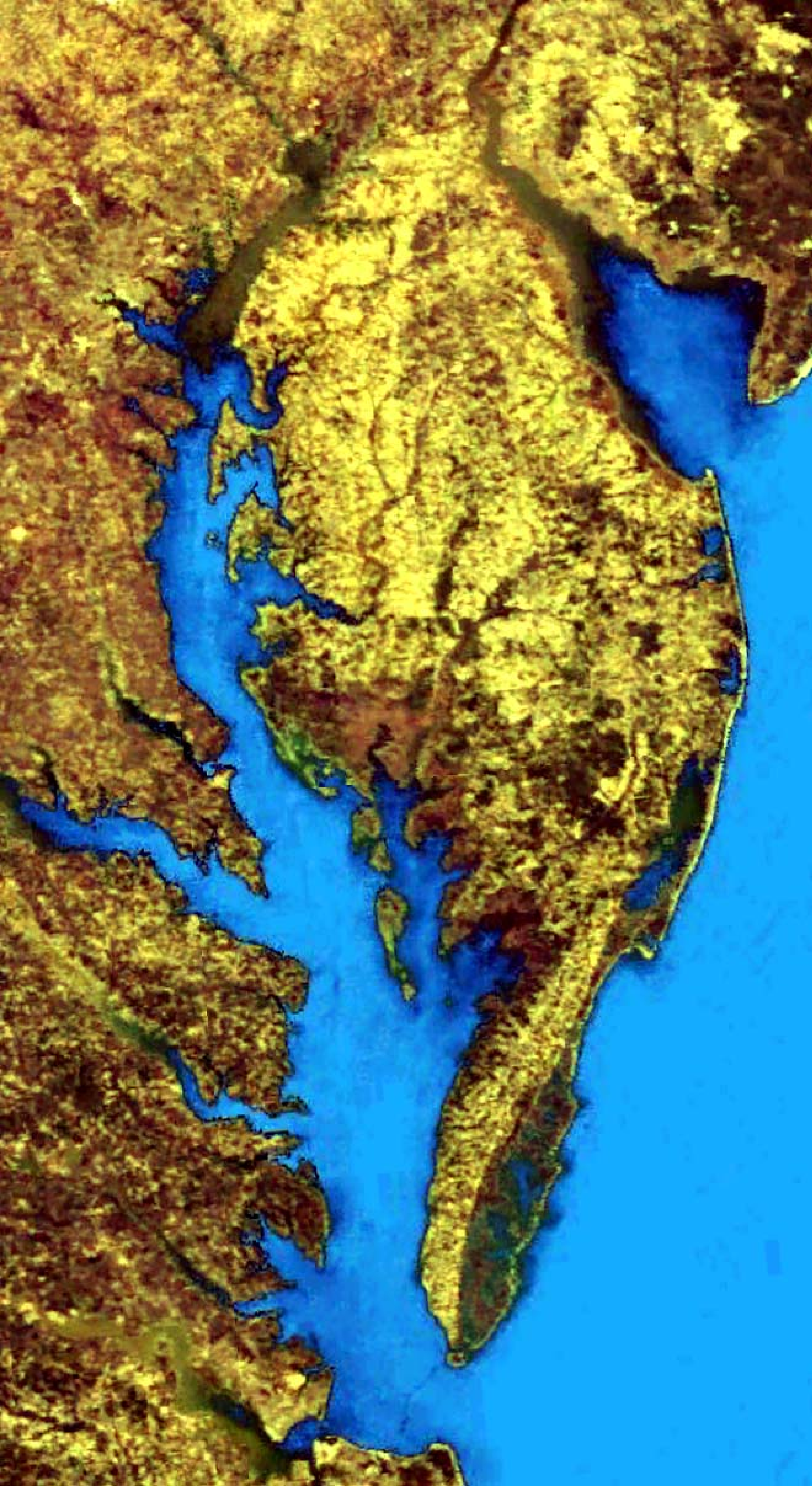
http://web.usna.navy.mil/~rmm/



From 2004 - Present:

- USNA has begun studies specific to the Chesapeake Bay
- Current efforts span five departments:
- Physics, Math, Chemistry, Oceanography and Naval Architecture
- Efforts have begun to join CBOS
- Instrumentation has begun in the Severn River and College Creek
- Study of small estuaries -"feeder" systems - to the Bay started
- Three Trident scholars: Gillary and Brasher (2004), Boe (2006)
- Differing approaches taken: Normal Mode Analysis,
- Navier-Stokes integration, COMSOL model
- 100 modes calculated for Dirichlet and Neumann (2005)
- Feasibility study for Galerkin method to extract $\overrightarrow{u}(x,t)$(2006)
- Initial Value Problem/ Dual Time Problem / Dual Position Problem
- $\sim$ 10 monitoring stations needed for $\overrightarrow{u}(x_i,t)$
- Concerns about proper mesh for Bay
- How to handle varying wave speed in the Bay?

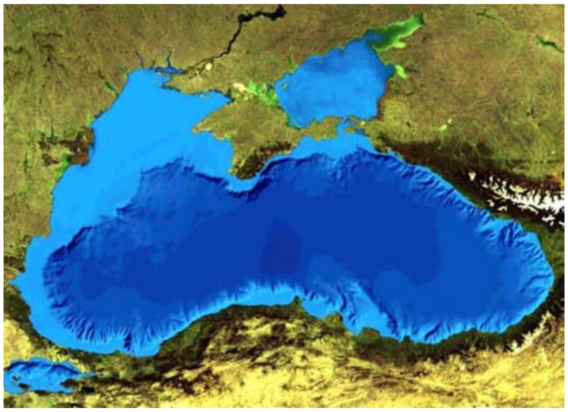


Fig. 1 The Black Sea compliments of NASA World Wind.

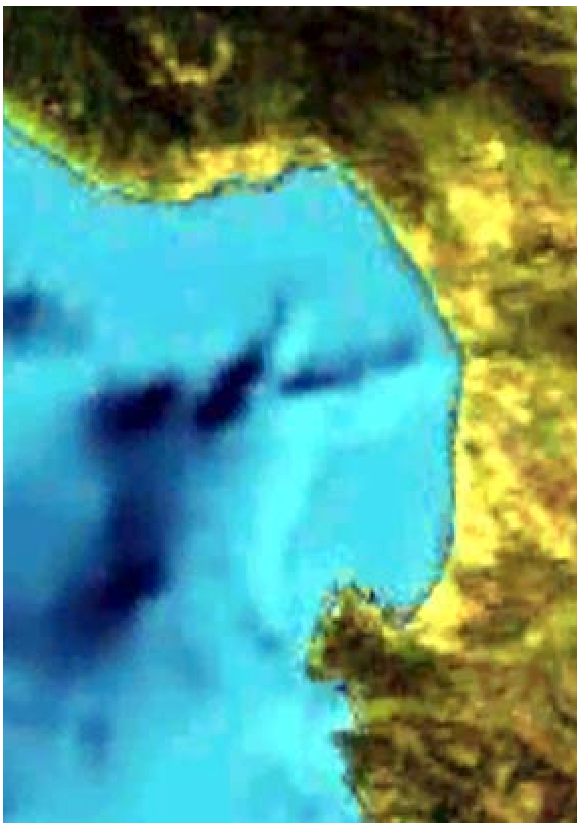


Fig. 2 Monterey Bay from NASA World Wind. The open boundary with the Pacific clearly visible.

and 5 illustrate the varying levels of complexity that have been solved using NMA.

The basic unit of calculation used throughout this paper is the normal mode. Like the modes of a guitar string or an organ pipe, systems obeying the Helmholtz equation and Dirichlet or Neumann boundary conditions will resonate in states referred as "normal modes". For the Chesapeake Bay, the modes calculated are energy potentials whose gradients and curls of gradients correspond to the vector current fields found in fluid mechanics ($\overrightarrow{u}$).

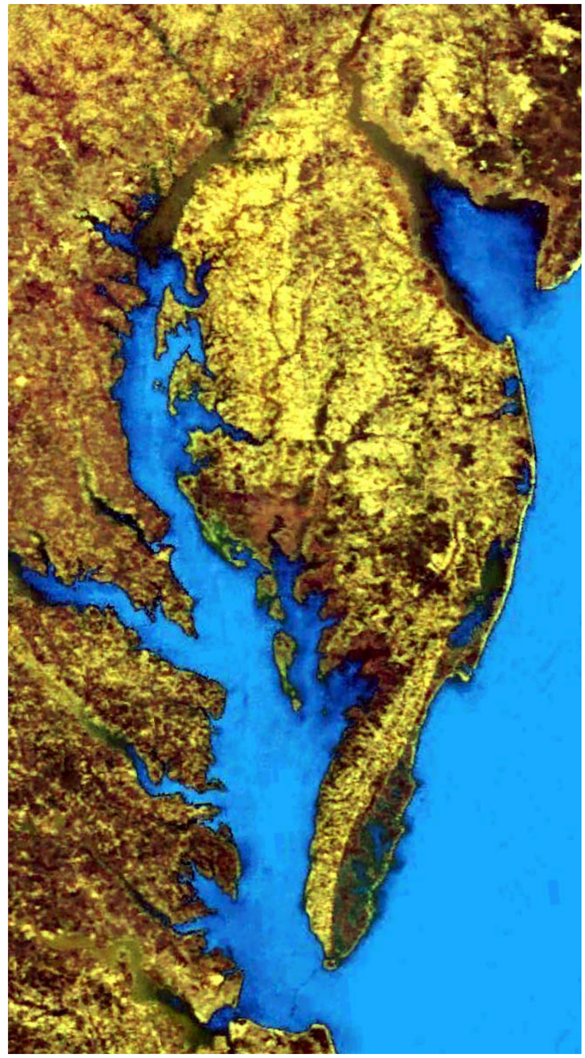


Fig. 3 Chesapeake Bay from NASA World Wind. The Atlantic ocean open at the southern end, allowing in salt water. The north end dominated by fresh water.

Briefly, the formulation leading to the calculation of fluid flow stems from the realization that the vector fields can be derived from two scalar fields, which are the solutions to the Helmhotz equation under Dirichlet and Neumann boundary conditions [4].

$$\overrightarrow{u} = \nabla \times [(\hat{n}\Psi) + \nabla \times (\hat{n}\Phi)]. \tag{1}$$

Here Ψ is the stream potential where,

$$\overrightarrow{u}_D = (u,v)_D = \left(\frac{-\partial\Psi}{\partial y}, \frac{\partial\Psi}{\partial x}\right), \tag{2}$$

and Φ is the velocity potential where,

$$\overrightarrow{u}_N = (u,v)_N = \left(\frac{\partial\Phi}{\partial x}, \frac{\partial\Phi}{\partial y}\right), \tag{3}$$

with (u,v) representing the surface current velocities in the x and y directions respectively. The total velocity field is composed:

Monterey Bay

- Lipphardt et al. at Univ. of Deleware have continued work on Monterey Bay, developing near real-time “Nowcasts”.
- http://newark.cms.udel.edu/~brucel/slmaps/
- http://newark.cms.udel.edu/~brucel/realtimemaps

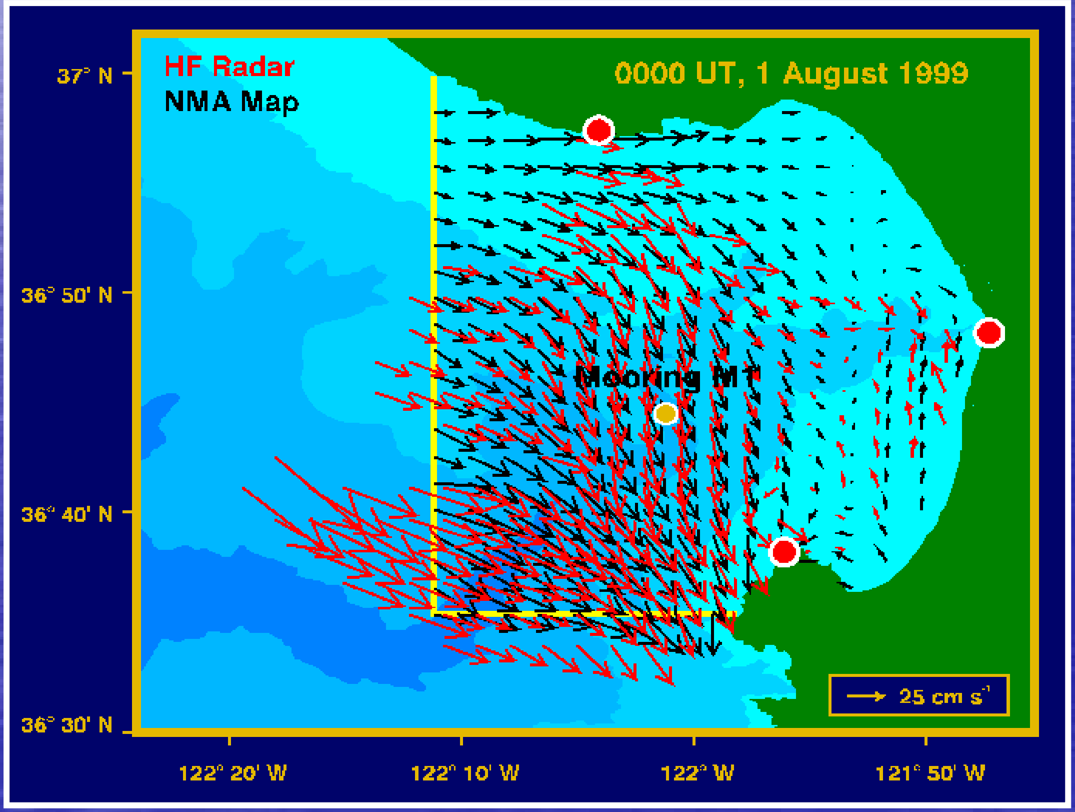
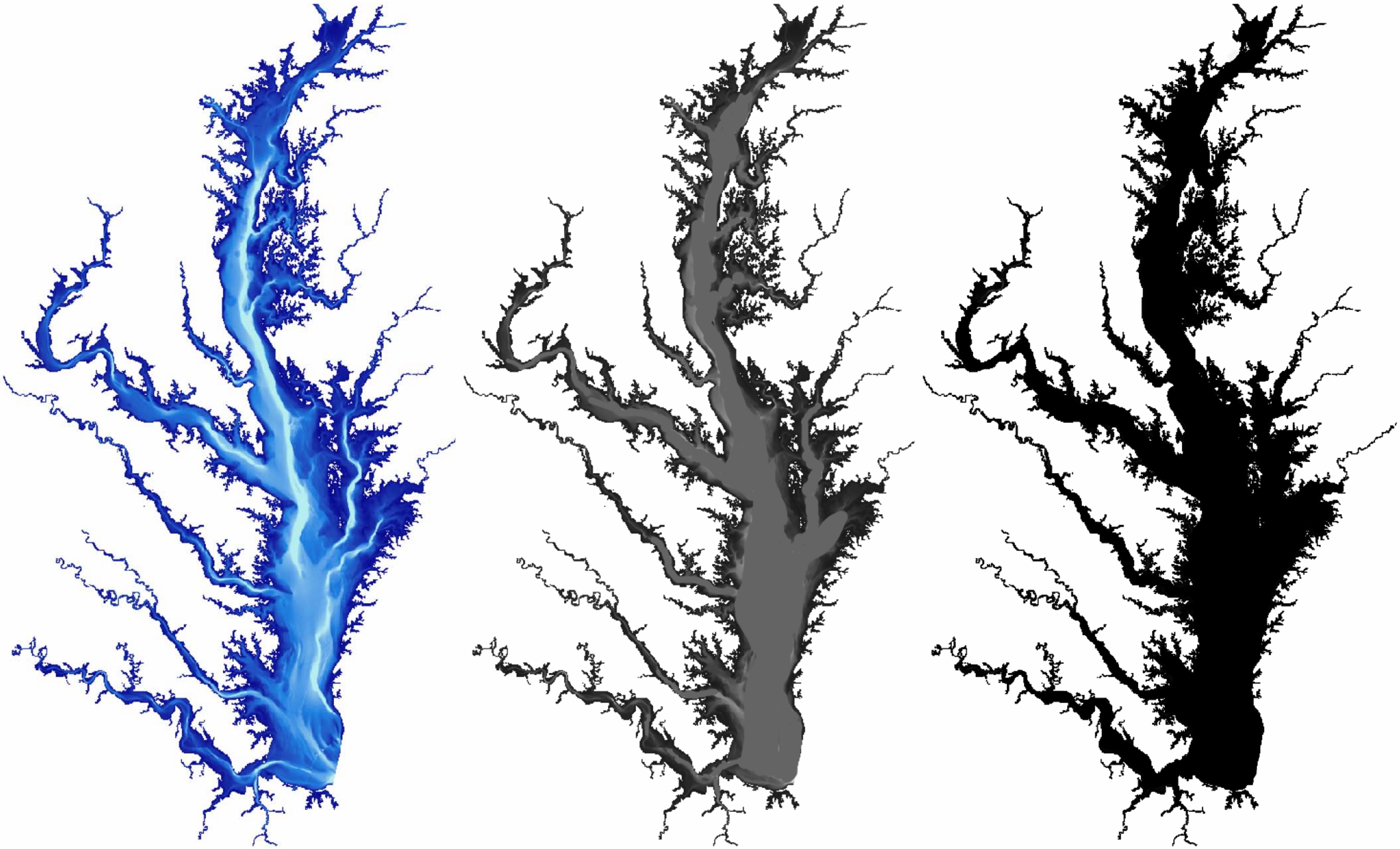
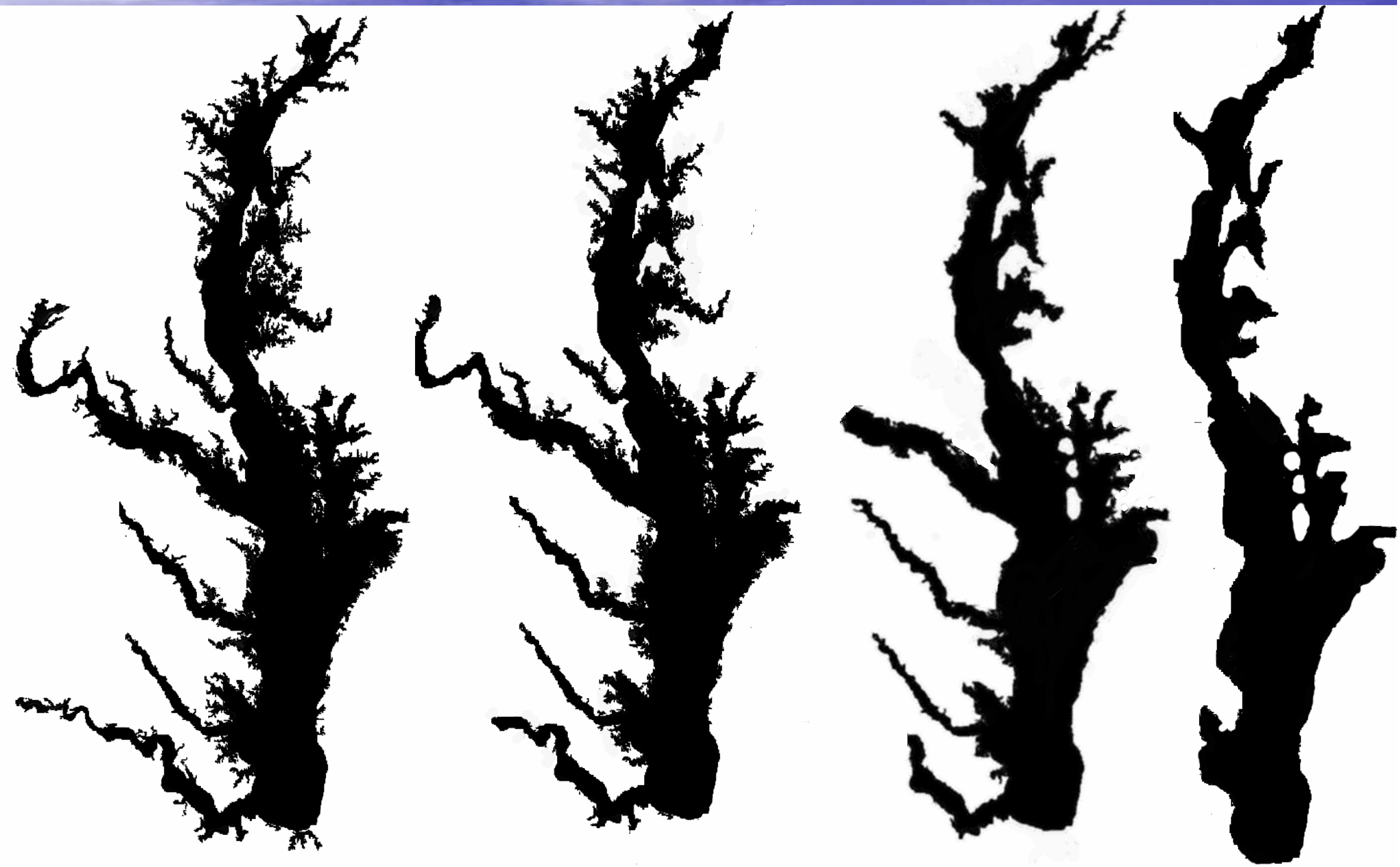


Image Processing of the Chesapeake

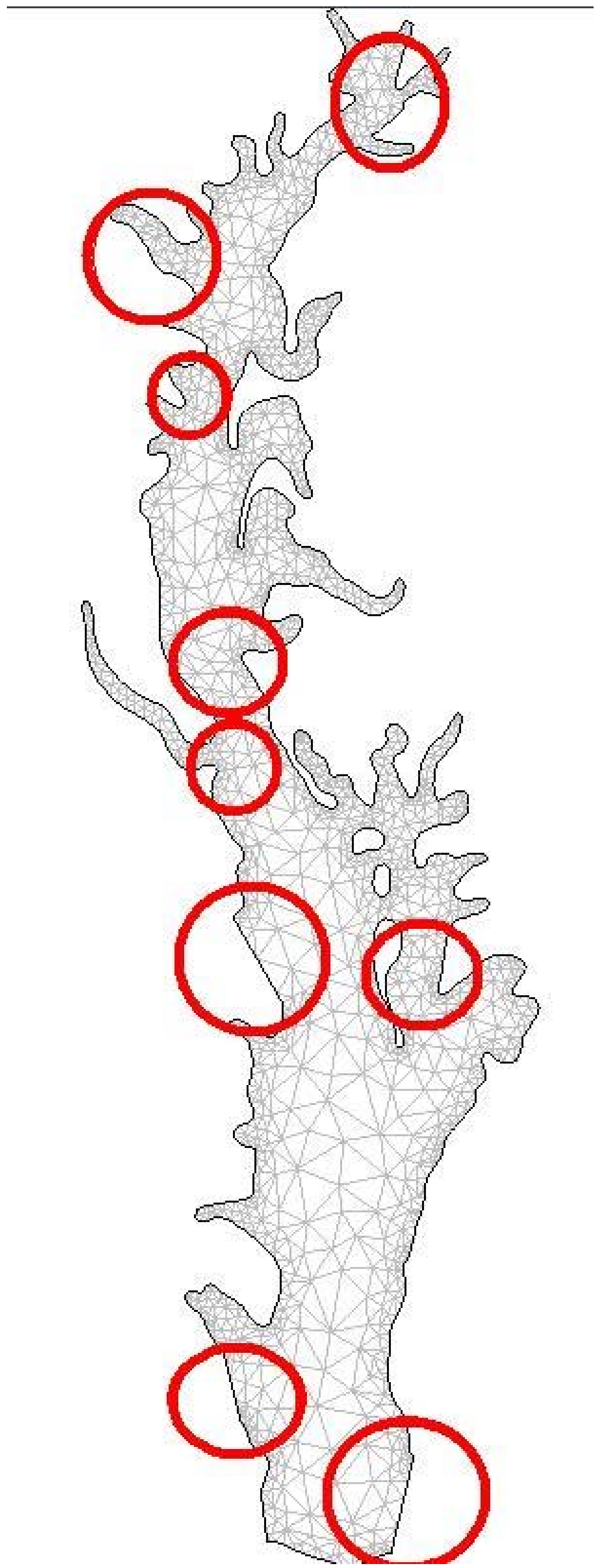


Approximated Boundaries



Galerkin Method:

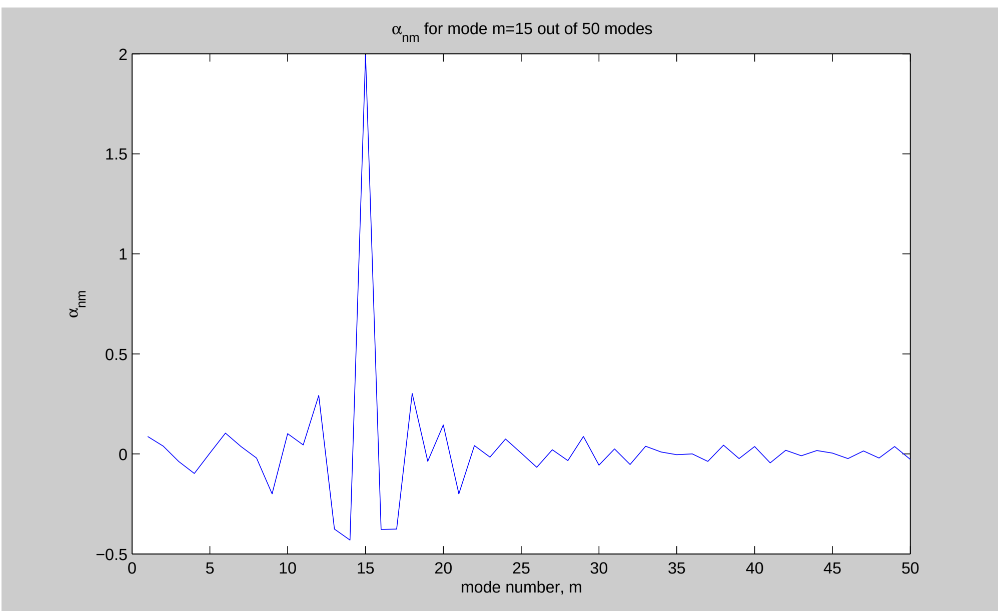
$$
\begin{aligned}
u(x,t) &= \sum_{n=0}^{\infty}\left[a(t)_n u(x)_{D,n} + b(t)_n u(x)_{N,n}\right] \\
a(t)_m &= \oint u(x,t)_{data} u(x)_{D,m} d\Omega \\
a(t)_m &= \oint \sum_{n=0}^{\infty}\left[a(t)_n u(x)_{D,n} + b(t)_n u(x)_{N,n}\right] u(x)_{D,m} d\Omega \\
a(t)_m &= \sum_{n=0}^{\infty}\left[a(t)_n \oint u_{D,n} u_{D,m} d\Omega + b(t)_n \oint u_{N,n} u_{D,m} d\Omega\right] \\
a(t)_m &= \sum_{n=0}^{\infty}\left[a(t)_n \delta_{nm} + b(t)_n \emptyset\right] \\
a(t)_m &= \delta_{nm} a(t)_n . \\
a(t)_n &= \oint u(x,t)_{data} u(x)_{D,n} d\Omega
\end{aligned}
$$



Partial Galerkin Method:

$$
\begin{aligned}
\tilde{a}(t)_m &= \int_{\tilde{\Omega}} u(\overrightarrow{x}, t) u(\overrightarrow{x})_{D,n} d\Omega \\
\tilde{a}(t)_m &= \int_{\tilde{\Omega}} \sum_{n=0}^{\infty} \left[a(t)_n u(\overrightarrow{x})_{D,n} + b(t)_n u(\overrightarrow{x})_{N,n} \right] u(\overrightarrow{x})_{D,n} d\Omega \\
\tilde{a}(t)_m &= \sum_{n=0}^{\infty} \left[a(t)_n \int_{\tilde{\Omega}} u_{D,n} u_{D,m} d\Omega + b(t)_n \int_{\tilde{\Omega}} u_{N,n} u_{D,m} d\Omega \right] \qquad (1) \\
\tilde{a}(t)_m &= \sum_{n=0}^{\infty} \left[\alpha_{D,nm} a(t)_n + \beta_{D,nm} b(t)_n \right]
\end{aligned}
$$

Note that the $\alpha_{D,nm}$ and $\beta_{D,nm}$ exhibit wavelet-like responses in the spectral domain.



Dual Time Problem (Initial Value Problem)

$$
\begin{aligned}
F(x,t)_{0,n} &= f(x)_n g(t)_n \\
F(x,t)_{data} &= \sum_{n=0}^{\infty} \left[A_n g(t)_{D,n} + B_n g(t)_{N,n} \right] \left[C_n f(x)_{D,n} + D_n f(x)_{N,n} \right] \\
F(x,t)_{data} &= \sum_{n=0}^{\infty} \left[AC_n g_{D,n} f_{D,n} + BC_n g_{N,n} f_{D,n} + AD_n g_{D,n} f_{N,n} + BD_n g_{N,n} f_{N,n} \right] \\
\alpha(t_1)_m &= \oint F(x,t_1)_{data} f(x)_{D,m} d\Omega \\
\alpha(t_1)_m &= \oint \sum_{n=0}^{\infty} \left[AC_n g_{D,n} f_{D,n} + BC_n g_{N,n} f_{D,n} + AD_n g_{D,n} f_{N,n} + BD_n g_{N,n} f_{N,n} \right] f(\\
\alpha(t_1)_n &= AC_n g(t_1)_{D,n} + BC_n g(t_1)_{N,n}
\end{aligned}
$$

$$
\begin{aligned}
\alpha(t_1)_m &= \oint F(x,t_1)_{data} f(x)_{D,m} d\Omega \\
\beta(t_1)_m &= \oint F(x,t_1)_{data} f(x)_{N,m} d\Omega \\
\gamma(t_2)_m &= \oint F(x,t_2)_{data} f(x)_{D,m} d\Omega \\
\Delta(t_2)_m &= \oint F(x,t_2)_{data} f(x)_{N,m} d\Omega \\
\alpha(t_1)_n &= AC_n g(t_1)_{D,n} + BC_n g(t_1)_{N,n} \\
\beta(t_1)_n &= AD_n g(t_1)_{D,n} + BD_n g(t_1)_{N,n} \\
\gamma(t_2)_n &= AC_n g(t_2)_{D,n} + BC_n g(t_2)_{N,n} \\
\Delta(t_2)_n &= AD_n g(t_2)_{D,n} + BD_n g(t_2)_{N,n}
\end{aligned}
$$

$$\begin{pmatrix} \alpha_n & \gamma_n \\ \beta_n & \Delta_n \end{pmatrix} = \begin{pmatrix} AC_n & BC_n \\ AD_n & BD_n \end{pmatrix} \begin{pmatrix} g(t_1)_{D,n} & g(t_2)_{D,n} \\ g(t_1)_{N,n} & g(t_2)_{N,n} \end{pmatrix}$$

$$\begin{pmatrix} AC_n & BC_n \\ AD_n & BD_n \end{pmatrix} = \begin{pmatrix} \alpha_n & \gamma_n \\ \beta_n & \Delta_n \end{pmatrix} \begin{pmatrix} g(t_1)_{D,n} & g(t_2)_{D,n} \\ g(t_1)_{N,n} & g(t_2)_{N,n} \end{pmatrix}^{-1}$$

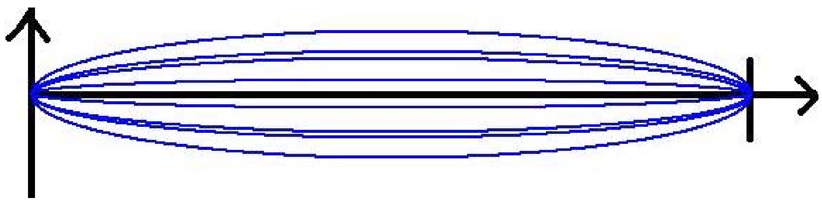
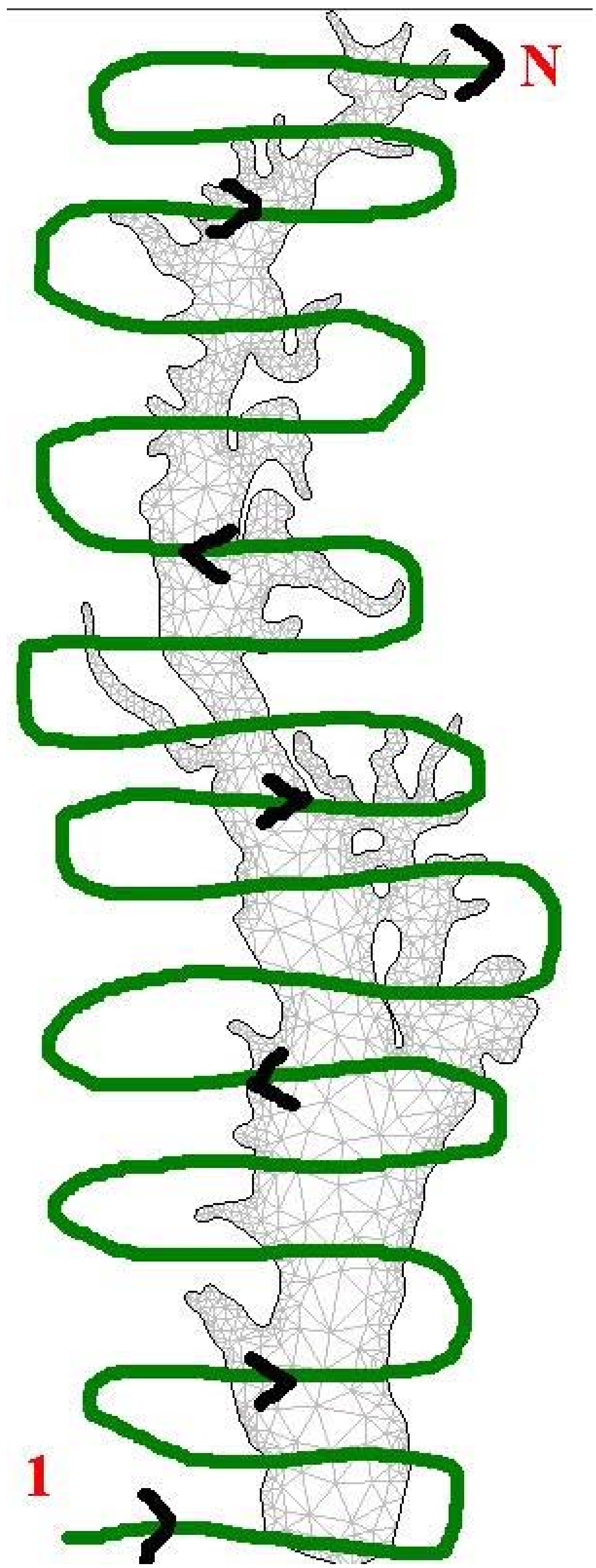


Figure 1: Guitar String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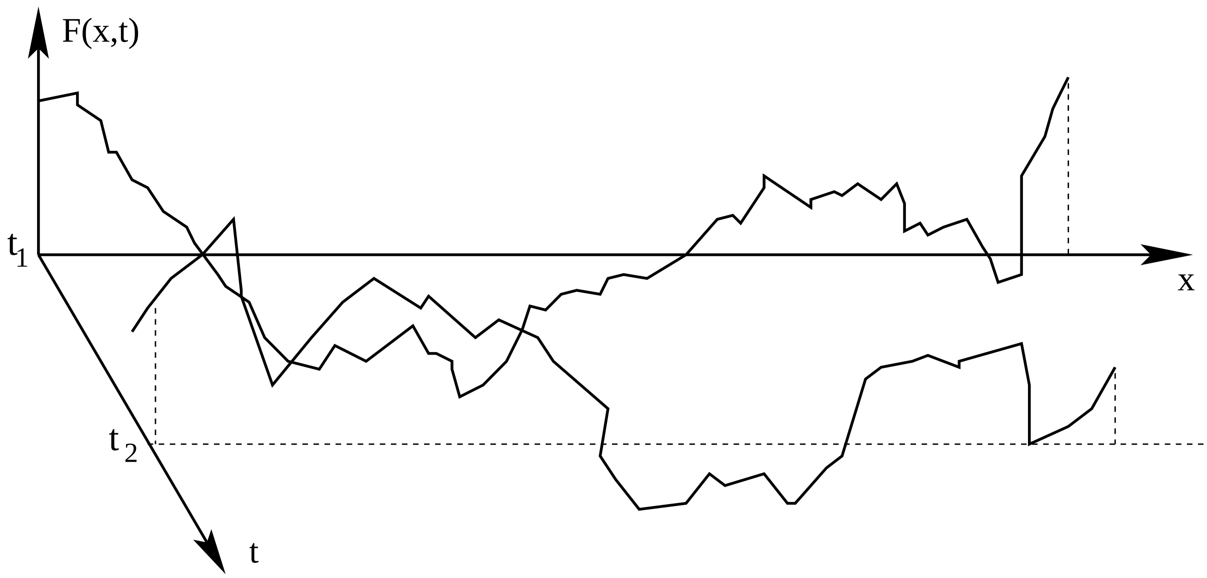


Figure 1: Dual Time Problem - similar to the Initial Value Problem

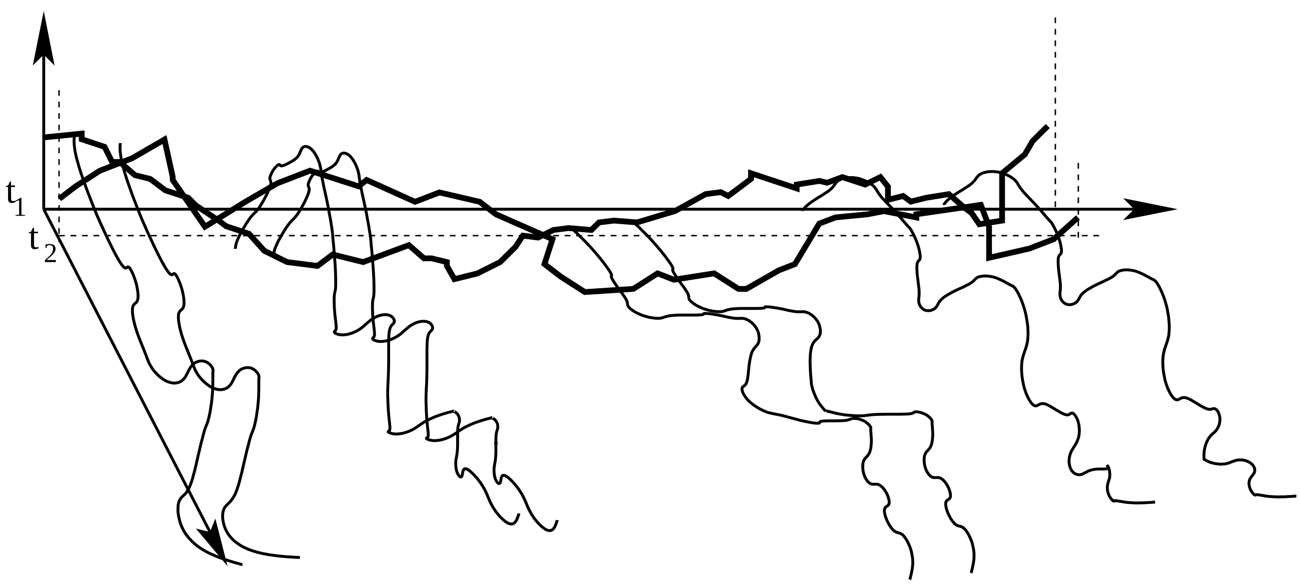


Figure 2: Having found the amplitudes, the solution is projected forward in time.

Dual Position Problem (conjugate to time problem)

$$
\begin{aligned}
F(x,t)_{0,n} &= f(x)_n g(t)_n \\
F(x,t)_{data} &= \sum_{n=0}^{\infty} \left[A_n g(t)_{D,n} + B_n g(t)_{N,n} \right] \left[C_n f(x)_{D,n} + D_n f(x)_{N,n} \right]
\end{aligned}
$$

$$
F(x,t)_{data} = \sum_{n=0}^{\infty} \left[AC_n g_{D,n} f_{D,n} + BC_n g_{N,n} f_{D,n} + AD_n g_{D,n} f_{N,n} + BD_n g_{N,n} f_{N,n} \right]
$$

$$
\alpha(x_1)_m = \int F(x_1,t)_{data} g(t)_{D,m} dt
$$

$$
\alpha(x_1)_m = \int \sum_{n=0}^{\infty} \Big[AC_n g_{D,n} f_{D,n} + BC_n g_{N,n} f_{D,n} + AD_n g_{D,n} f_{N,n} + BD_n g_{N,n} f_{N,n} \Big] g(
$$

$$
\alpha(x_1)_n = AC_n f(x_1)_{D,n} + AD_n f(x_1)_{N,n}
$$

$$
\begin{aligned}
\alpha(x_1)_m &= \int F(x_1,t)_{data} g(t)_{D,m} dt \\
\beta(x_1)_m &= \int F(x_1,t)_{data} g(t)_{N,m} dt \\
\gamma(x_2)_m &= \int F(x_2,t)_{data} g(t)_{D,m} dt \\
\Delta(x_2)_m &= \int F(x_2,t)_{data} g(t)_{N,m} dt \\
\alpha(x_1)_n &= AC_n g(t_1)_{D,n} + AD_n g(t_1)_{N,n} \\
\beta(x_1)_n &= BC_n g(t_1)_{D,n} + BD_n g(t_1)_{N,n} \\
\gamma(x_2)_n &= AC_n g(t_2)_{D,n} + AD_n g(t_2)_{N,n} \\
\Delta(x_2)_n &= BC_n g(t_2)_{D,n} + BD_n g(t_2)_{N,n}
\end{aligned}
$$

$$\begin{pmatrix} \alpha_n & \gamma_n \\ \beta_n & \Delta_n \end{pmatrix} = \begin{pmatrix} AC_n & AD_n \\ BC_n & BD_n \end{pmatrix} \begin{pmatrix} f(x_1)_{D,n} & f(x_2)_{D,n} \\ f(x_1)_{N,n} & f(x_2)_{N,n} \end{pmatrix}$$

$$\begin{pmatrix} AC_n & AD_n \\ BC_n & BD_n \end{pmatrix} = \begin{pmatrix} \alpha_n & \gamma_n \\ \beta_n & \Delta_n \end{pmatrix} \begin{pmatrix} f(x_1)_{D,n} & f(x_2)_{D,n} \\ f(x_1)_{N,n} & f(x_2)_{N,n} \end{pmatrix}^{-1}$$

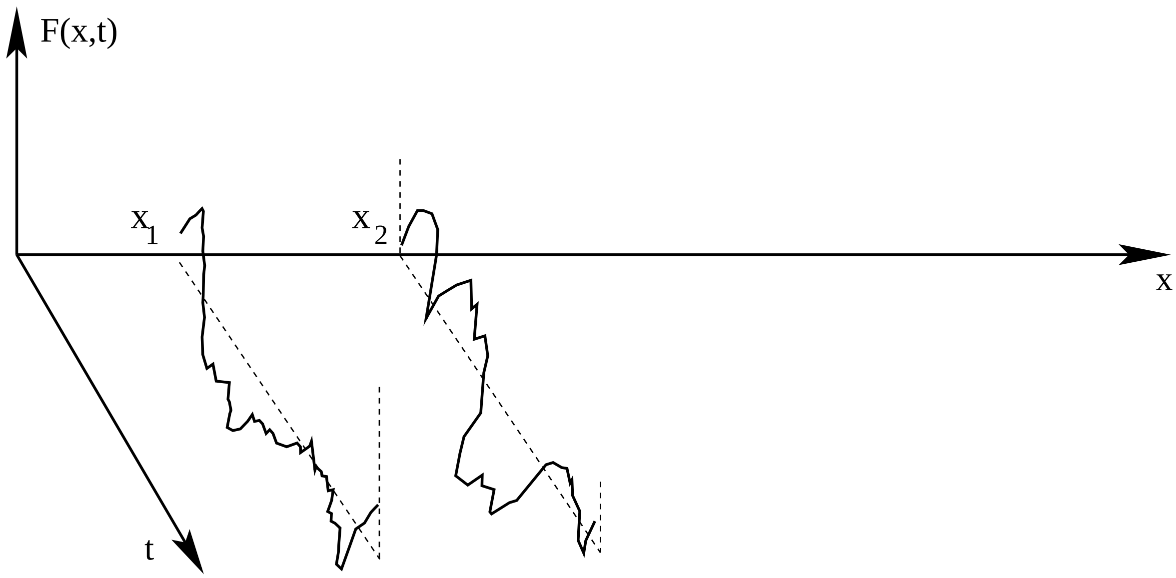


Figure 3: Dual Position Problem - conjugate to the Dual Time Problem

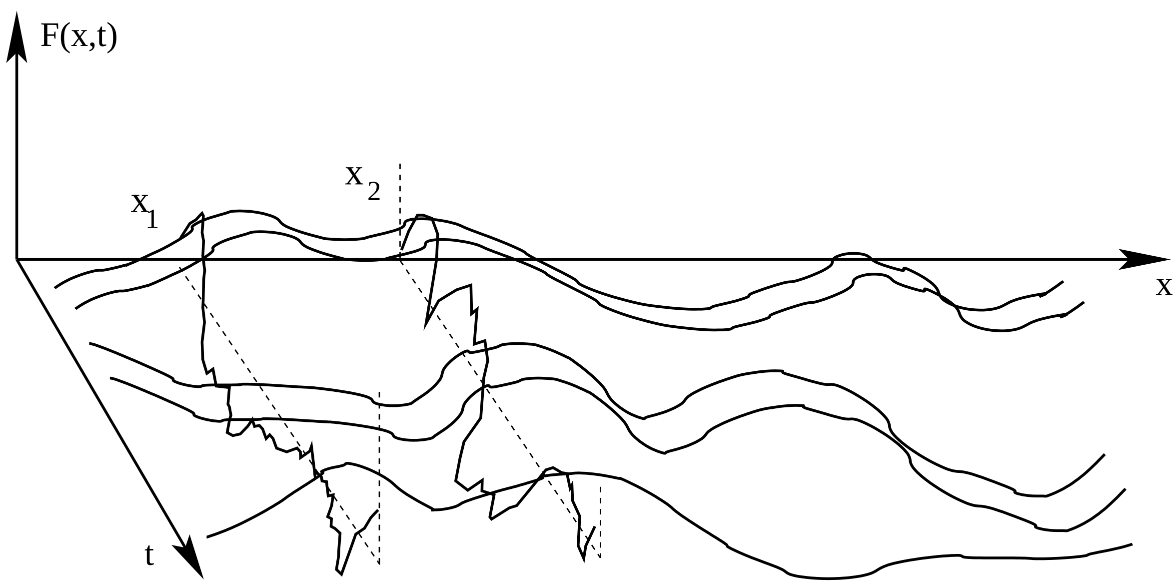


Figure 4: Having found the amplitudes, the solution is projected across the spatial domain.

Multiple Position Problem

$$
\begin{aligned}
F(x,t)_{0,n} &= f(x)_n g(t)_n \\
F(x,t)_{data} &= \sum_{n=0}^{\infty} \left[A_n g(t)_{D,n} + B_n g(t)_{N,n} + E_n t + F_n\right]\left[C_n f(x)_{D,n} + D_n f(x)_{N,n} + G_n x + H_n\right] \\
F(x,t)_{data} &= \sum_{n=0}^{\infty} \left[AC_n g_{D,n} f_{D,n} + BC_n g_{N,n} f_{D,n} + AD_n g_{D,n} f_{N,n} + BD_n g_{N,n} f_{N,n}\right] \\
F(x,t)_{data} &= \ldots \;\; 16 \quad terms, \quad needs \;\; 8 \;\; locations
\end{aligned}
$$

$$
\begin{pmatrix} 8x8 & \\ & \end{pmatrix} = \ldots
$$

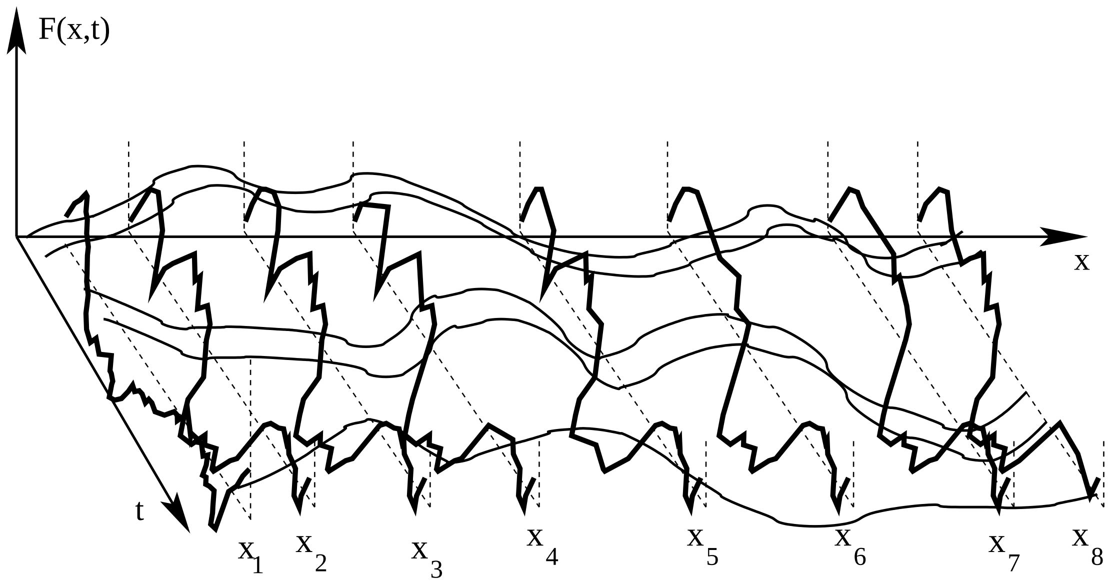
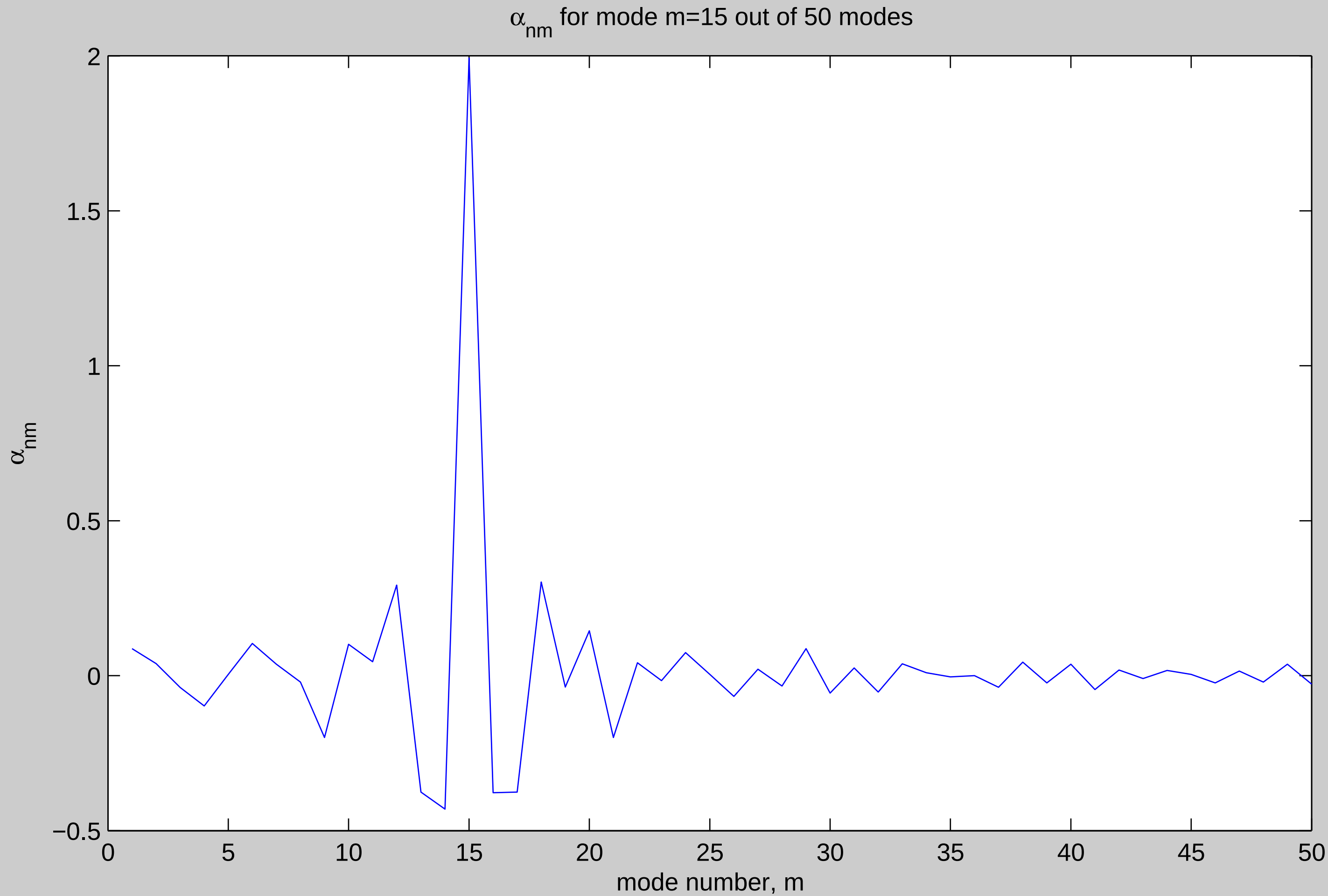
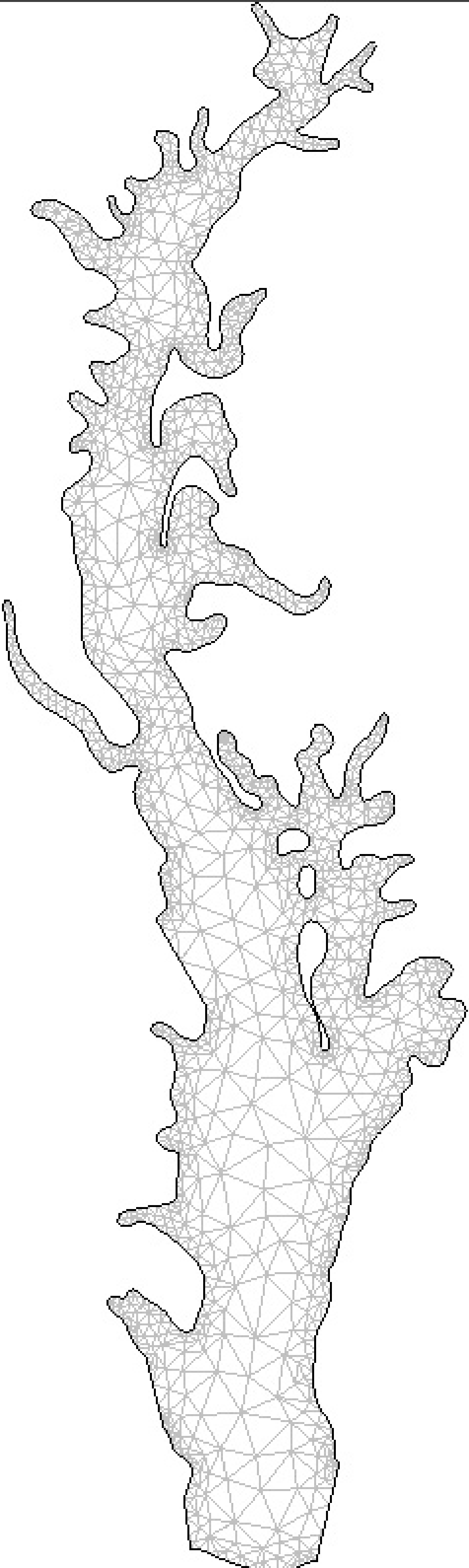
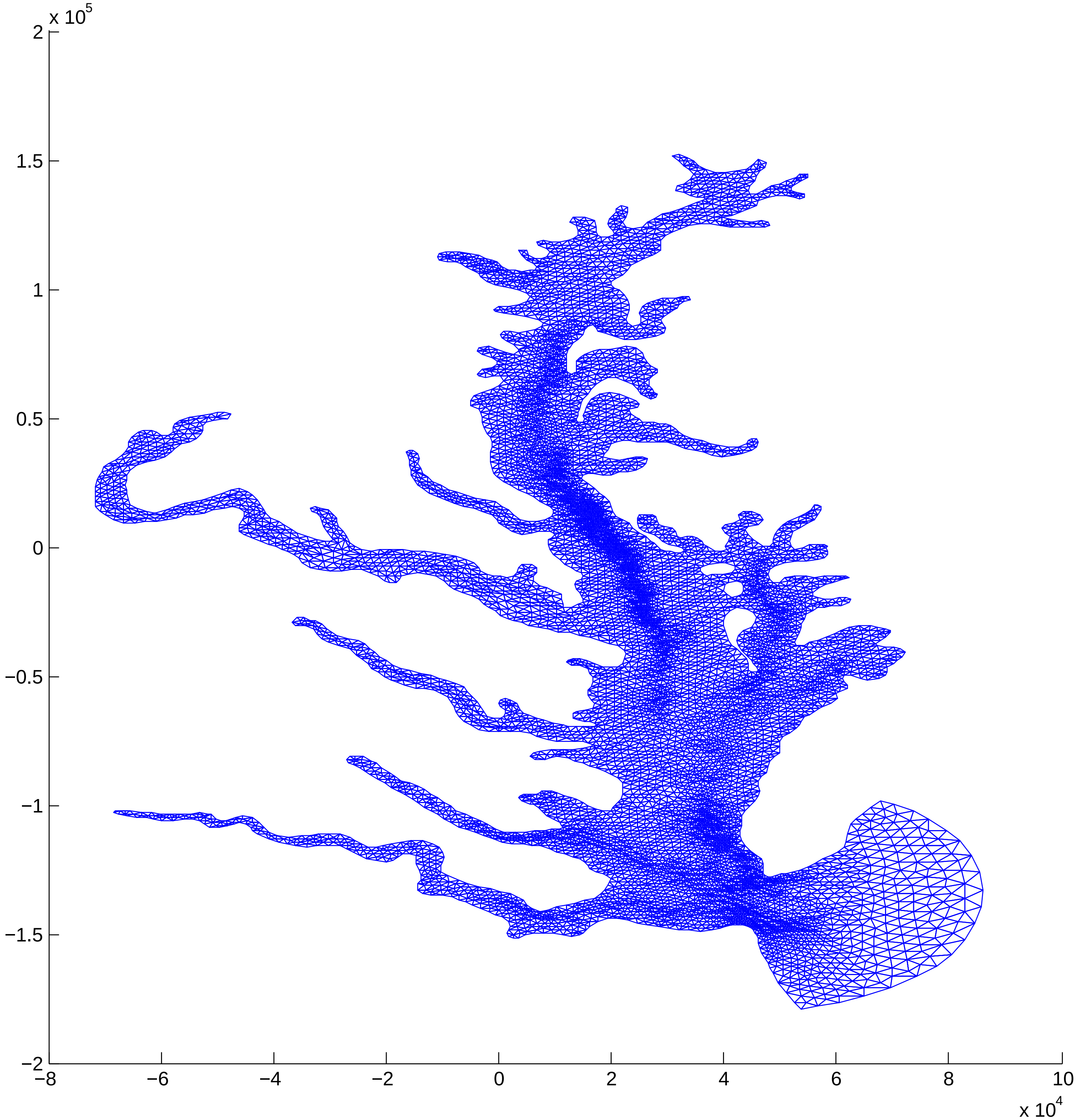


Figure 5: Having found the amplitudes, the solution is projected across the spatial domain.

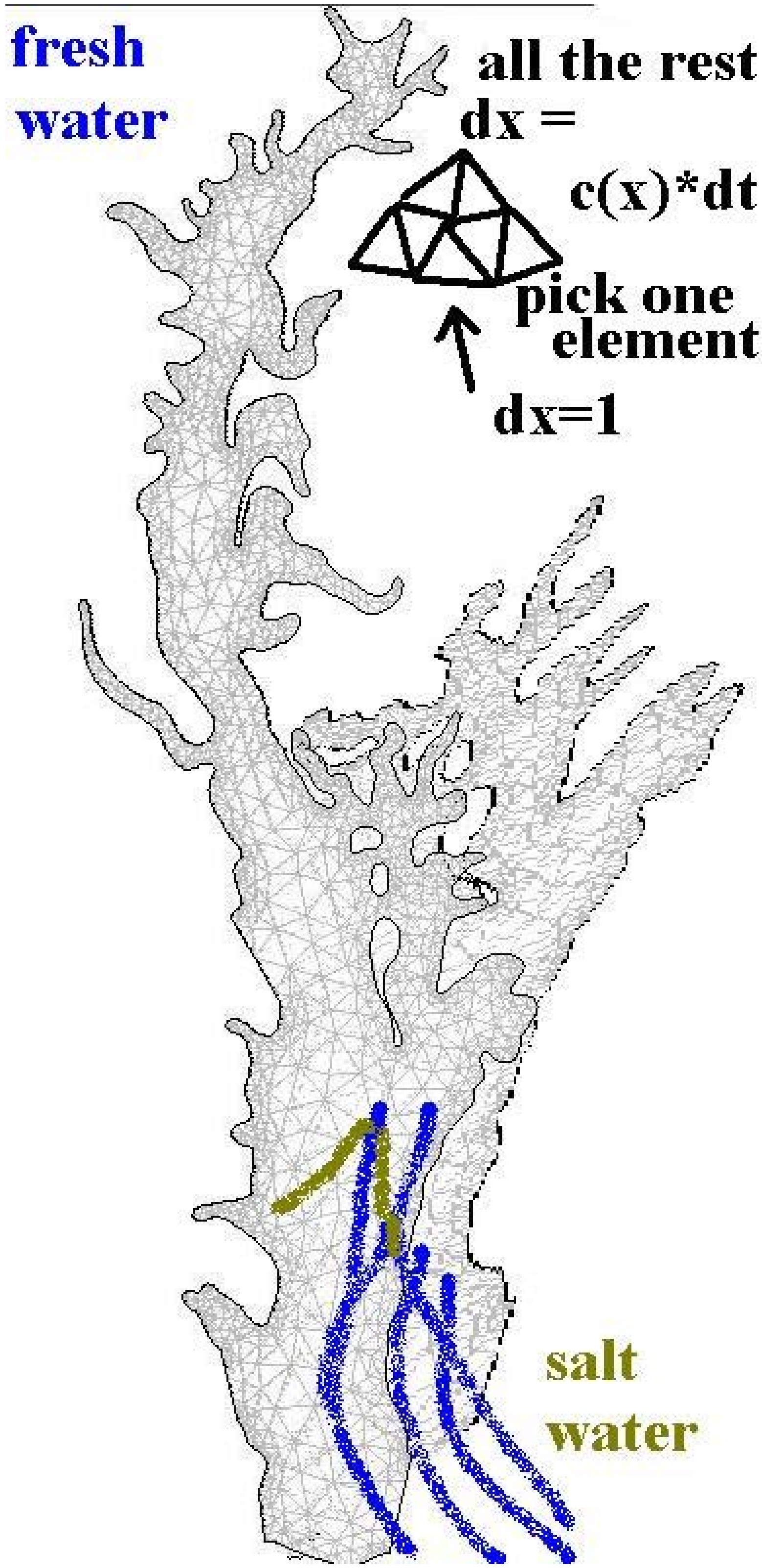


αnm for mode m=15 out of 50 modes
αnm
mode number, m
2
1.5
1
0.5
0
−0.5
0
5
10
15
20
25
30
35
40
45
50

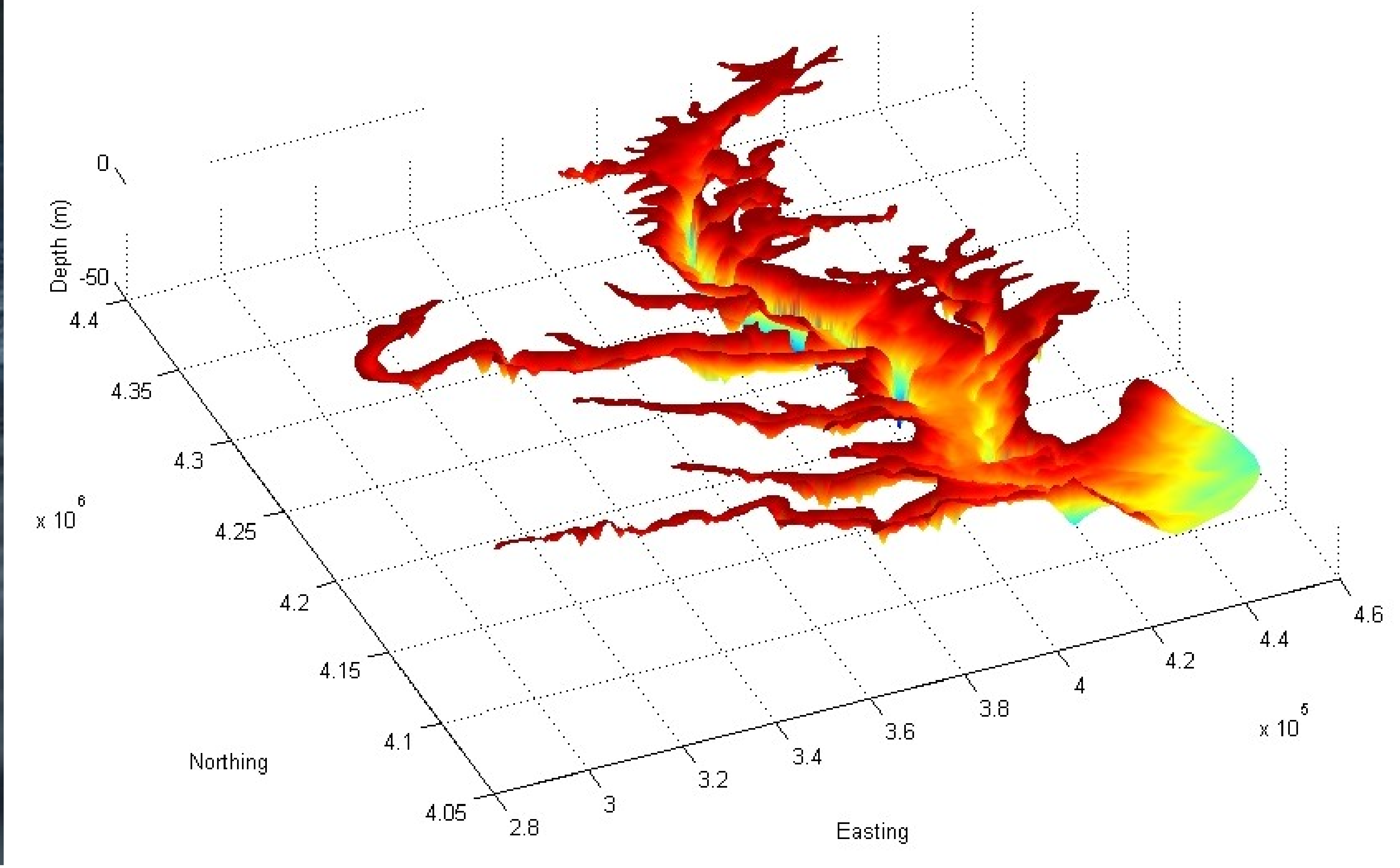




x 10^5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−8
−6
−4
−2
0
2
4
6
8
10
x 10^4


fresh
water
all the rest
dx =
c(x)*dt
pick one
element
dx=1
salt
water

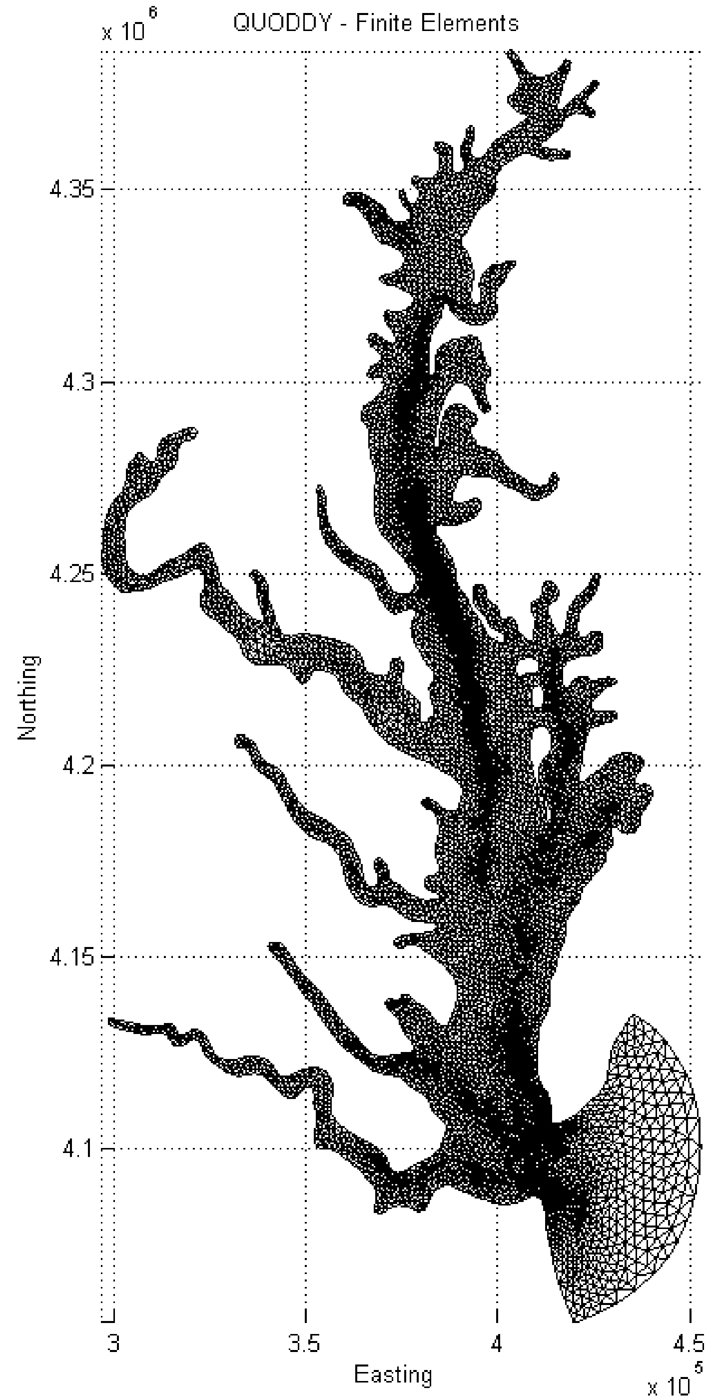
Bathymetry



0
-50
Depth (m)
4.4
4.35
4.3
4.25
4.2
4.15
4.1
4.05
x 10^6
Northing
2.8
3
3.2
3.4
3.6
3.8
4
4.2
4.4
4.6
x 10^5
Easting

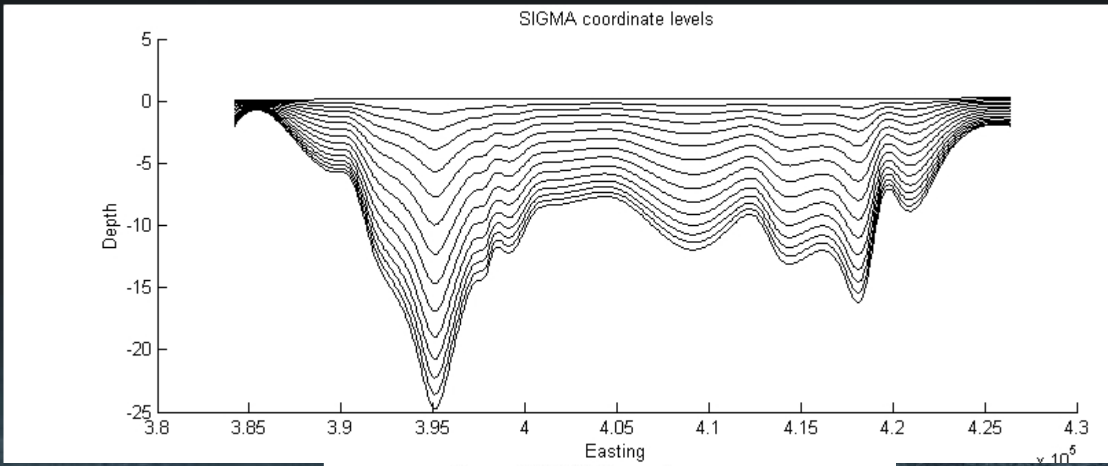
Chesapeake Bay Analysis

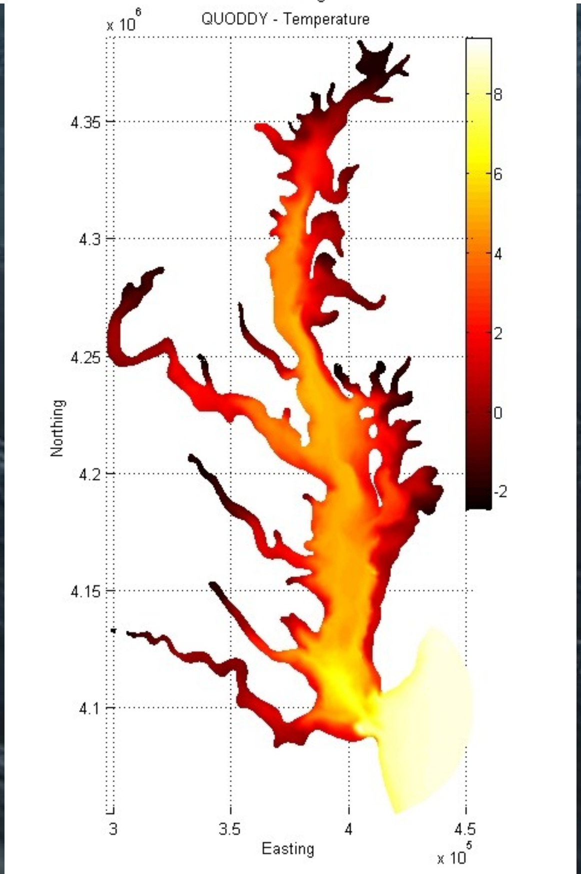
- QUODDY Computer Model
  - Finite-Element Model
  - Fully 3-Dimensional
  - 9700 nodes

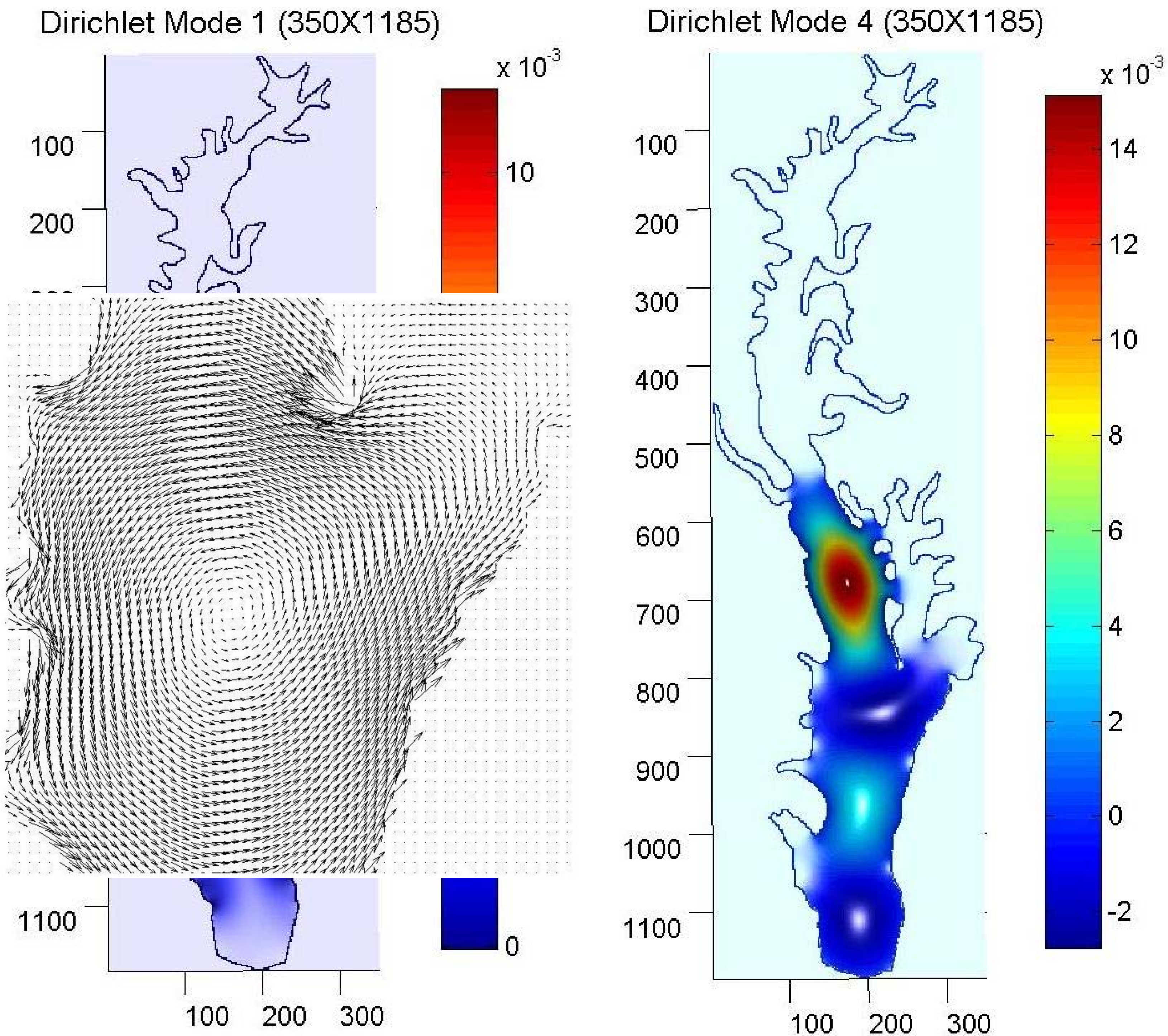


QUODDY

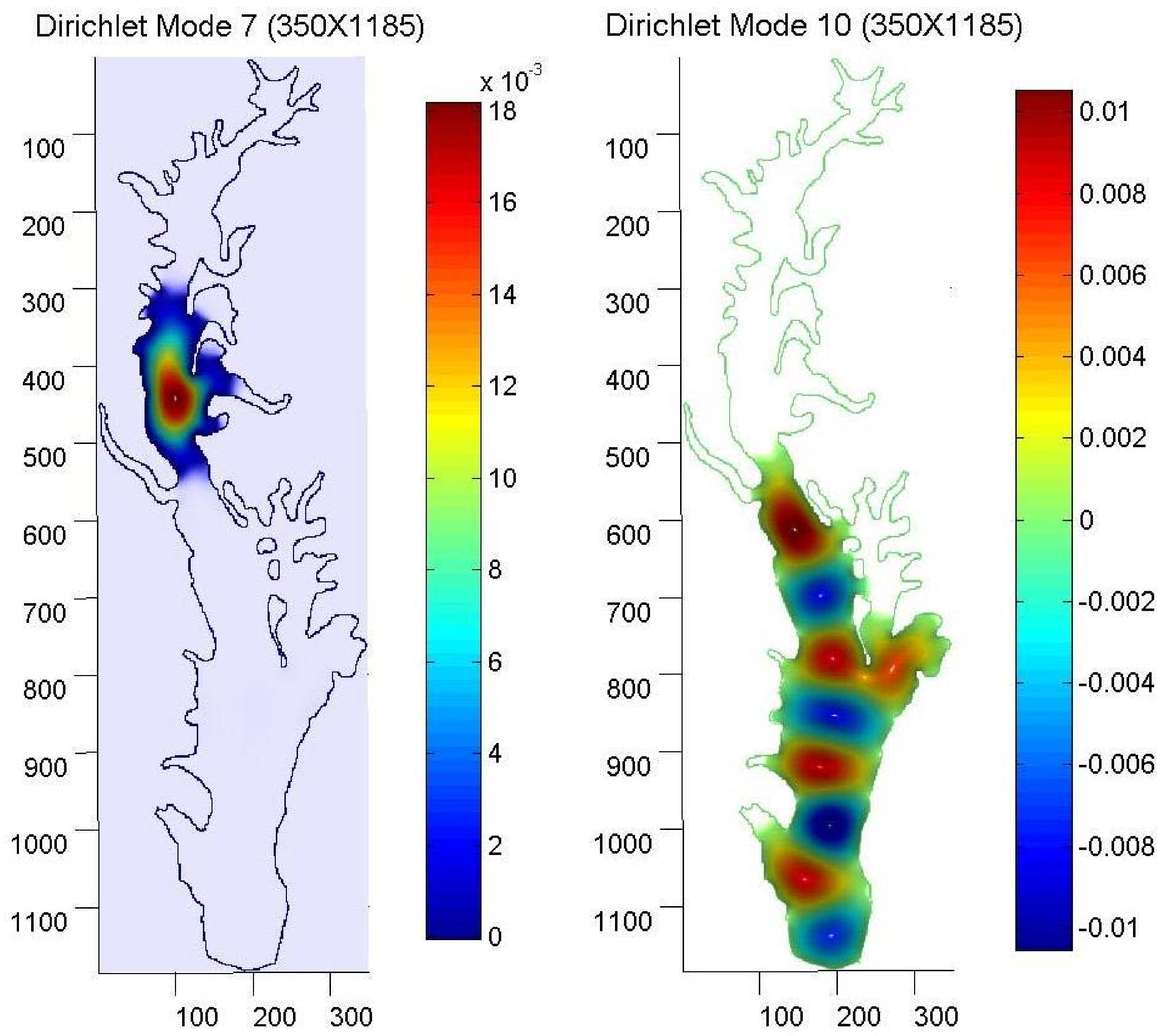
- Boussinesq Equations
  - Temperature
  - Salinity
- Sigma Coordinates
- No normal flow
- Winds, tides and river inflow included in model



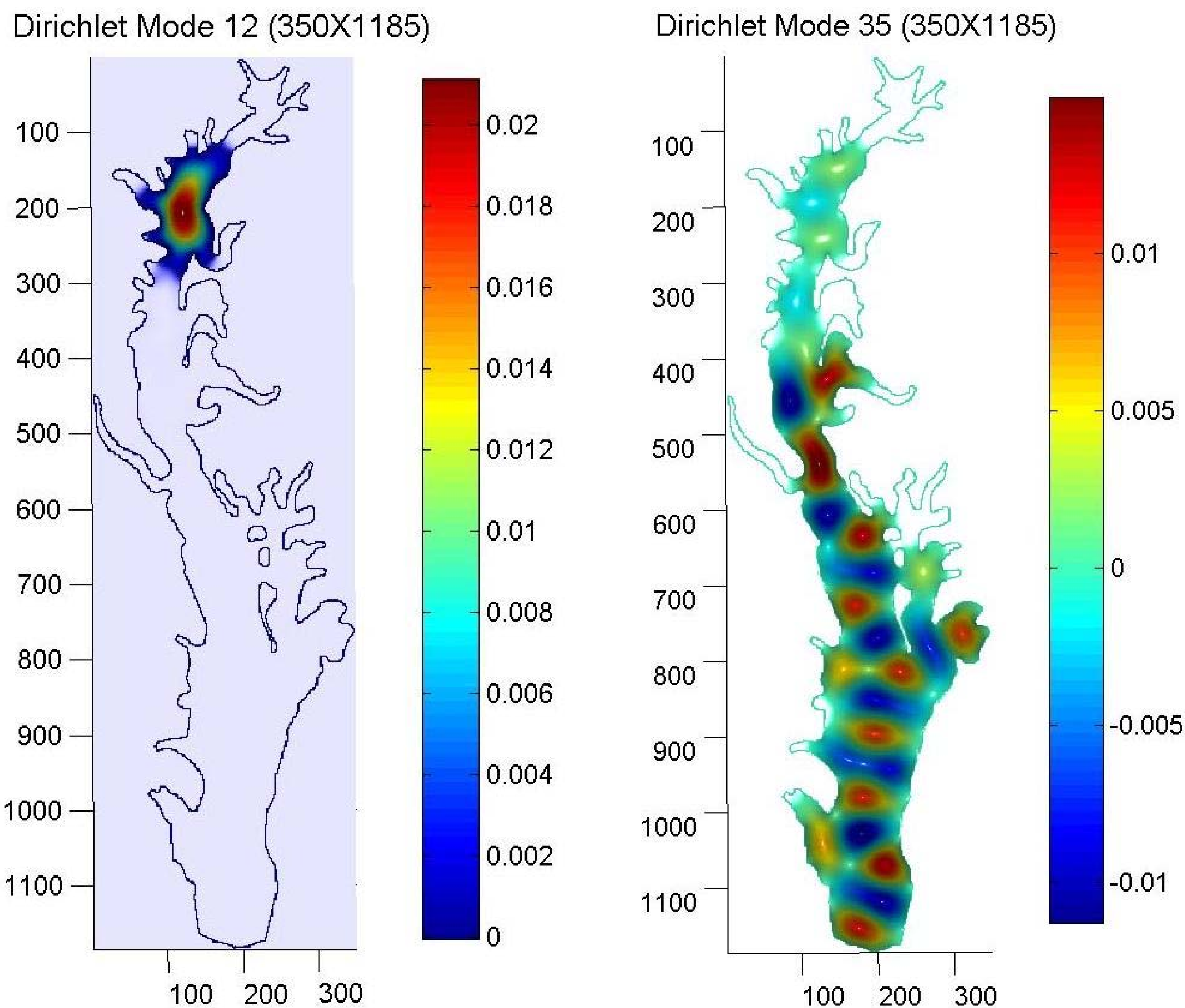




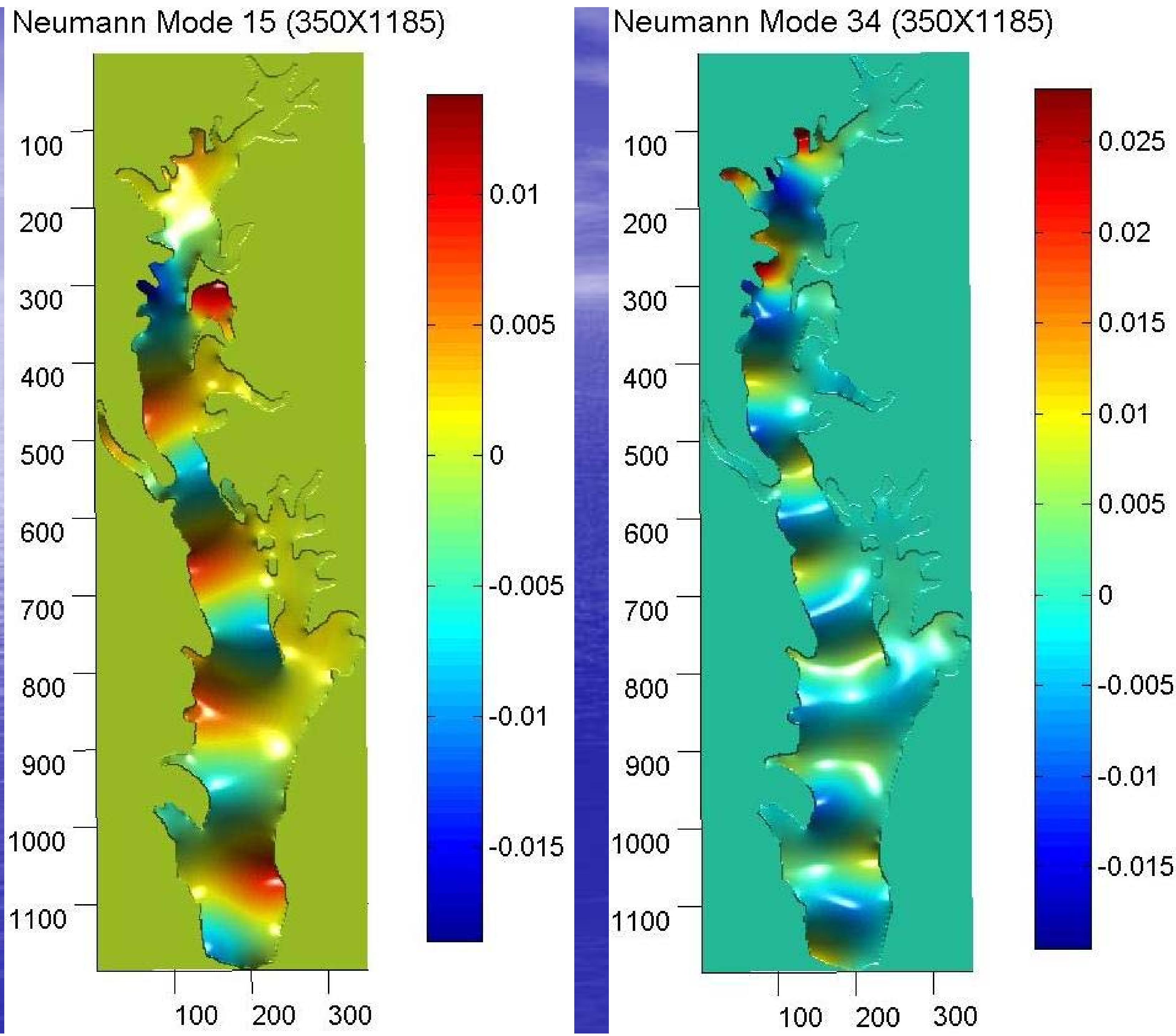
Dirichlet Mode 1 (350X1185)
x 10^-3
10
0
100
200
1100
100
200
300
Dirichlet Mode 4 (350X1185)
x 10^-3
14
12
10
8
6
4
2
0
-2
100
200
300
400
500
600
700
800
900
1000
1100
100
200
300



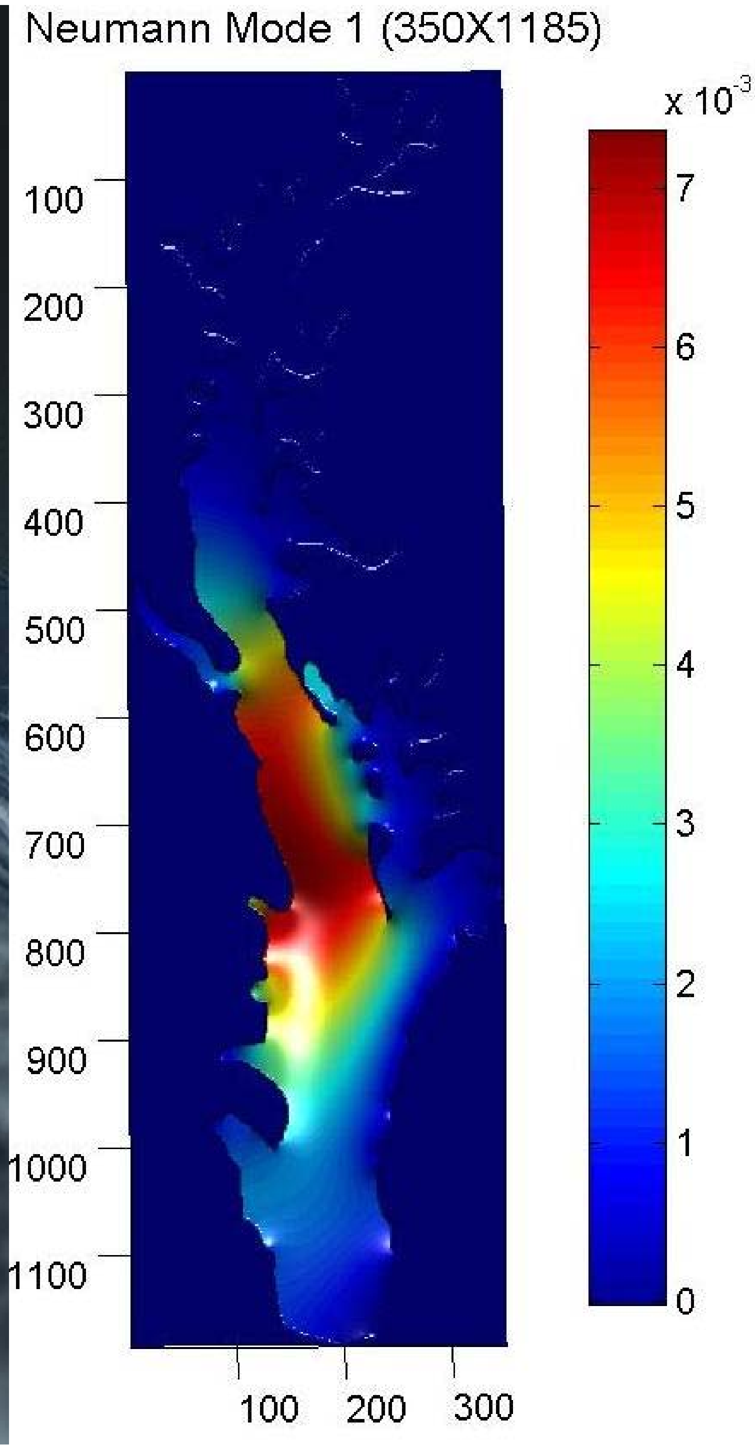
Dirichlet Mode 7 (350X1185)
Dirichlet Mode 10 (350X1185)
x 10^-3
18
16
14
12
10
8
6
4
2
0
0.01
0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
100
200
300
400
500
600
700
800
900
1000
1100
100
200
300

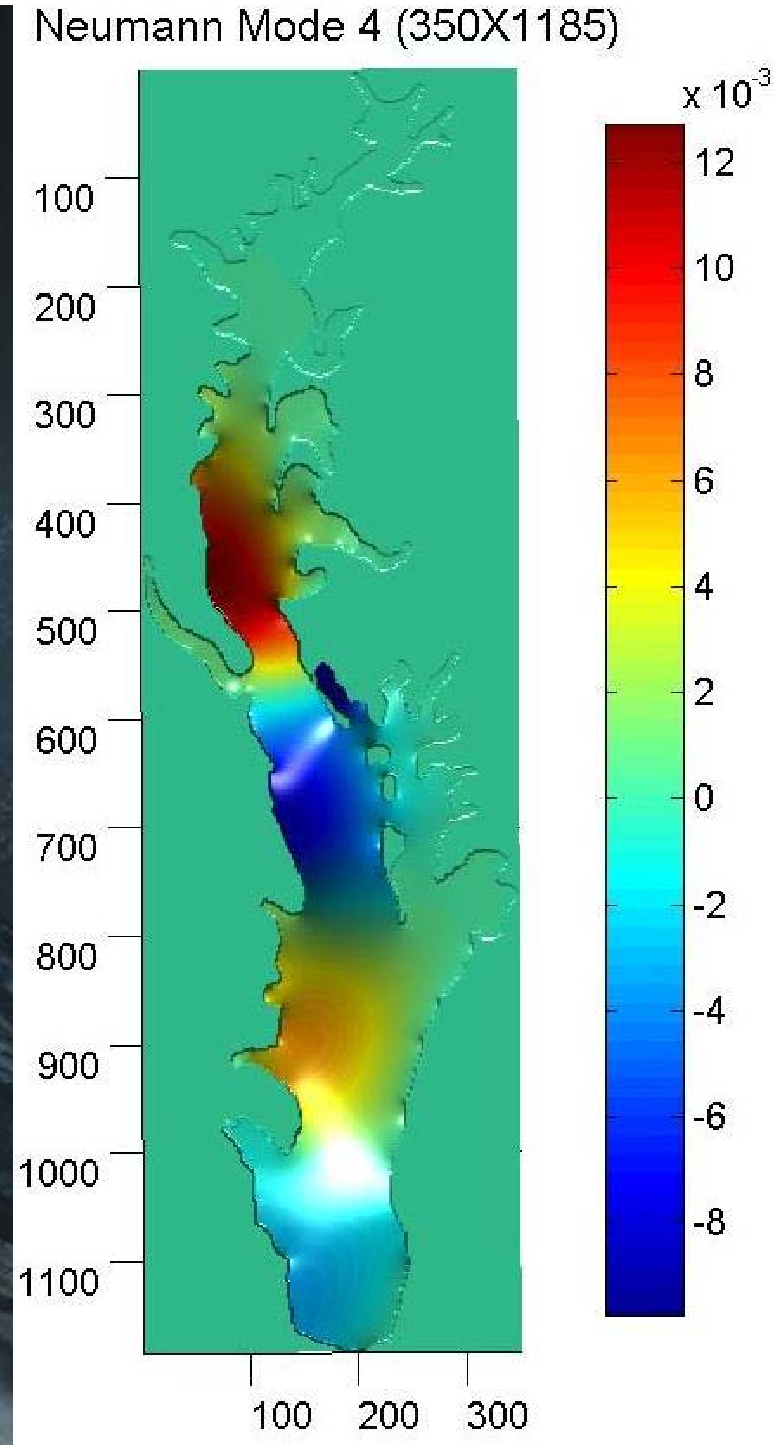


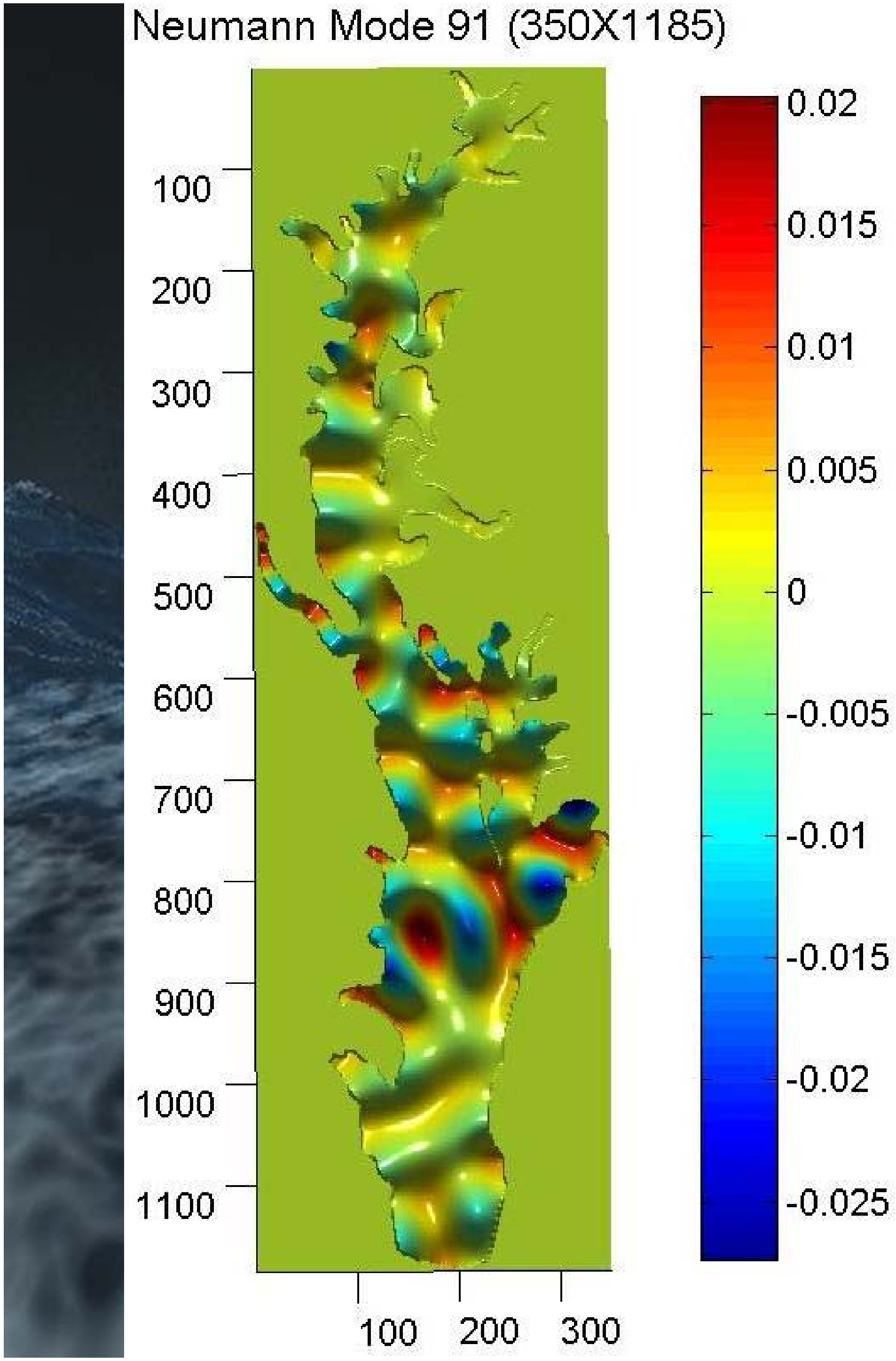
Dirichlet Mode 12 (350X1185)
100
200
300
400
500
600
700
800
900
1000
1100
100
200
300
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
Dirichlet Mode 35 (350X1185)
100
200
300
400
500
600
700
800
900
1000
1100
100
200
300
0.01
0.005
0
-0.005
-0.01

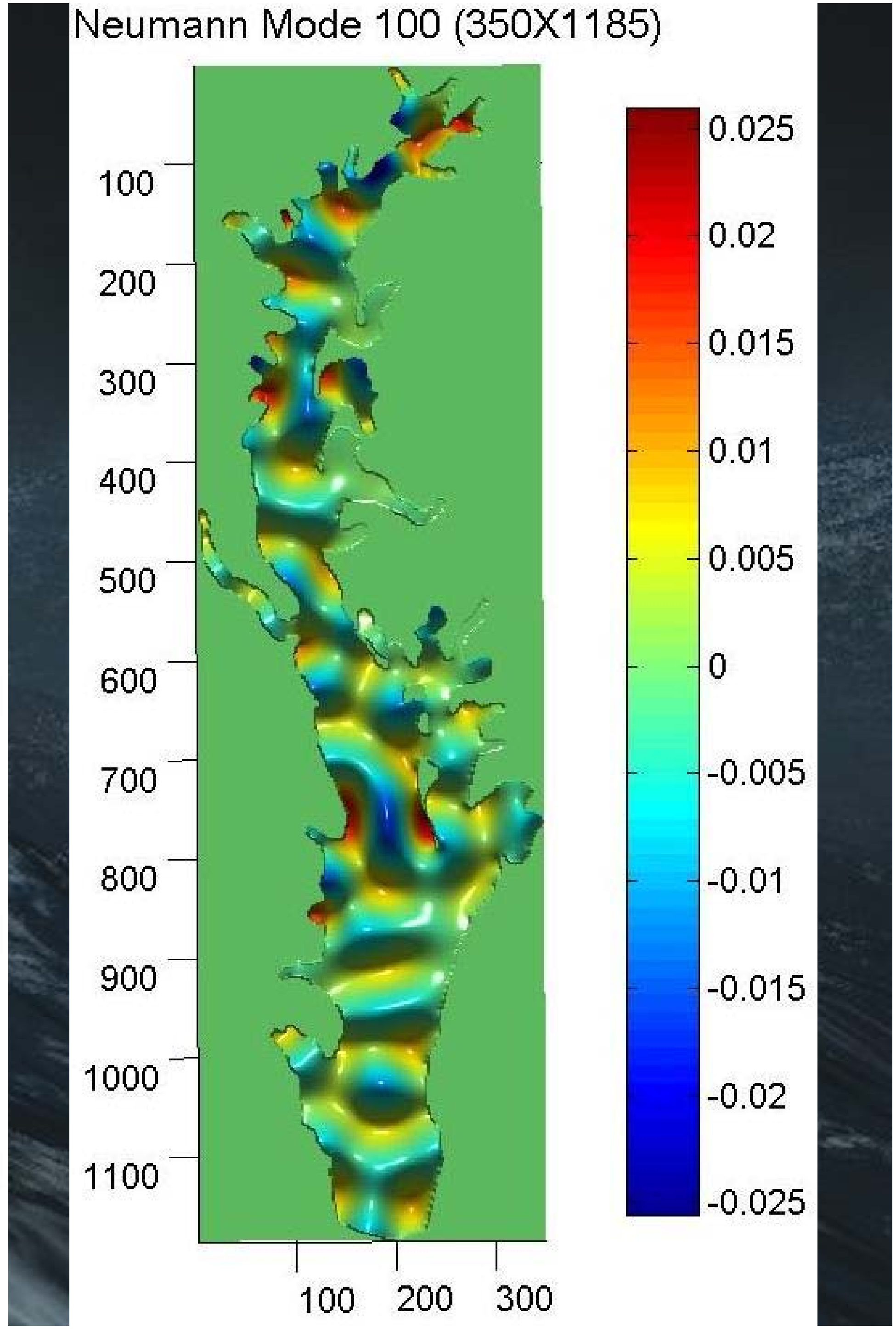


Neumann Mode 15 (350X1185)
Neumann Mode 34 (350X1185)
100
200
300
400
500
600
700
800
900
1000
1100
0.01
0.005
0
-0.005
-0.01
-0.015
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015


Neumann Mode 1 (350X1185)
x 10^-3
0
1
2
3
4
5
6
7
100
200
300
400
500
600
700
800
900
1000
1100


Neumann Mode 4 (350X1185)
x 10^-3
-8
-6
-4
-2
0
2
4
6
8
10
12
100
200
300
400
500
600
700
800
900
1000
1100


Neumann Mode 91 (350X1185)
100
200
300
400
500
600
700
800
900
1000
1100
100 200 300
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025


Neumann Mode 100 (350X1185)
100
200
300
400
500
600
700
800
900
1000
1100
100 200 300
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025

Methods of solution

- Zel'dovich (1985): Velocity vectors fields can be extracted from two scalar potentials

$$\vec{u} = \vec{\nabla} \times \left[\left(\hat{n}\Psi \right) + \vec{\nabla} \times \left(\hat{n}\Phi \right) \right]$$

$$\nabla^2 \Psi_n^D = -\lambda_n \Psi_n^D \qquad \Psi^D |_{boundary} = 0$$

$$\nabla^2 \Phi_m^N = -\mu_m \Phi_m^N \qquad (\hat{n} \cdot \vec{\nabla} \Phi^N) |_{boundary} = 0$$

- Lipphardt et al. (2000): Addition of forcing terms allows for non-conservation of mass through a boundary – ie. Water from rivers or the ocean is accounted.

$$\nabla^2 \Theta(x, y, 0, t) = S_\Theta(t)$$

$$(\hat{m} \cdot \vec{\nabla} \Theta) |_{boundary} = (\hat{m} \cdot \vec{u}_{\mathrm{model}}) |_{boundary}$$

Putting It All Together

- Ψ is the stream potential (vorticity mode).
- Φ is the velocity potential (divergent mode).
- Situation analogous to ($\vec{E}$,$\vec{B}$) fields from E&M.
- The vector field representation can be separated into two eigenvalue equations.
- Source term solved via Poisson's equation.
- The total vector field is written as a sum over all states for each representation.

$$(u,v) = \sum_{n=1}^{N} a_n \, (u_n, v_n)_D \; + \; \sum_{m=1}^{M} b_m \, (u_m, v_m)_N \; + \; (u(t), v(t))_S$$

Time Series Analysis

- Chesapeake flow can be written as a Normal Mode expansion => u(r,t)

$$\vec{u}(\vec{r},t) = \sum_{n=1}^{N} a_n(t)\,\vec{u}_{n,D}(\vec{r}) + b_n(t)\vec{u}_{n,N}(\vec{r}) + \vec{u}(\vec{r},t)_{src}$$

- Use Galerkin method to extract a(t), b(t)
- Due to limitations in data collection:
  - use QUODDY (model based on data)
  - Partial domain Galerkin method

Galerkin Method

$$f(\vec{r},t) = \sum_{n=1}^{N} a_n(t)\,\Psi_{n,D}(\vec{r}) + b_n(t)\Phi_{n,N}(\vec{r})$$

$$a_m = \oint \left\{ \sum_{n=1}^{N} a_n(t)\Psi_n(\vec{r})\cdot\Psi_m(\vec{r}) + b_n(t)\Phi_n(\vec{r})\cdot\Psi_m(\vec{r}) \right\} d\Omega_{full}$$

$$a_m = \sum_{n=1}^{N} a_n(t)\left\{ \oint \Psi_n(\vec{r})\cdot\Psi_m(\vec{r})\, d\Omega_{full} \right\} + \sum_{n=1}^{N} b_n(t)\left\{ \oint \Phi_n(\vec{r})\cdot\Psi_m(\vec{r})\, d\Omega_{full} \right\}$$

$$a_m = \sum_{n=1}^{N} a_n(t)\ \left\{ \delta_{nm} \right\} + \sum_{n=1}^{N} b_n(t)\ \left\{ 0 \right\}$$

$$a_m = a_n(t)$$

Partial Domain Galerkin Method

$$f(\vec{r},t) = \sum_{n=1}^{N} a_n(t)\,\Psi_{n,D}(\vec{r}) \;+\; b_n(t)\Phi_{n,N}(\vec{r})$$

$$\tilde{a}_m = \int \left\{ \sum_{n=1}^{N} a_n(t)\Psi_n(\vec{r})\cdot\Psi_m(\vec{r}) + b_n(t)\Phi_n(\vec{r})\cdot\Psi_m(\vec{r}) \right\} d\Omega_{partial}$$

$$\tilde{a}_m = \sum_{n=1}^{N} a_n(t)\left\{\int \Psi_n(\vec{r})\cdot\Psi_m(\vec{r})\,d\Omega_{partial}\right\} + \sum_{n=1}^{N} b_n(t)\left\{\int \Phi_n(\vec{r})\cdot\Psi_m(\vec{r})\,d\Omega_{partial}\right\}$$

$$let\;\; \alpha_{nm} = \int \Psi_n(\vec{r})\cdot\Psi_m(\vec{r})\; d\Omega_{partial}$$

$$let\;\; \beta_{nm} = \int \Phi_n(\vec{r})\cdot\Psi_m(\vec{r})\; d\Omega_{partial}$$

$$\tilde{a}_m = \sum_{n=1}^{N} \{\alpha_{nm}\}\, a_n(t) \;+\; \sum_{n=1}^{N} \{\beta_{nm}\}\, b_n(t)$$

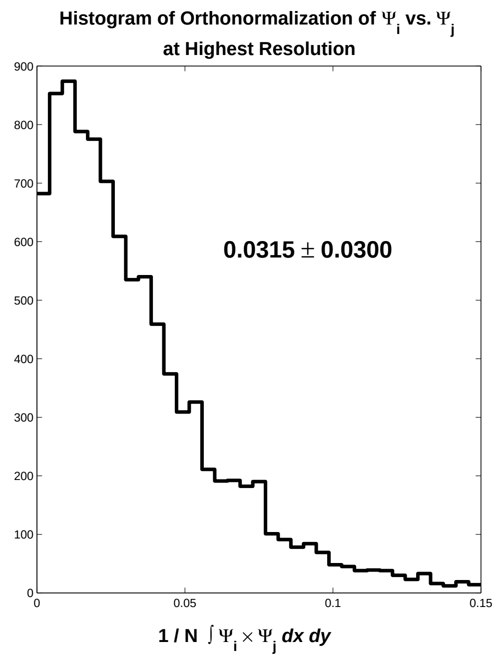


Fig. 6 Orthogonality between any two modes shows the degree which information about the system overlaps. For a basis set, the requirement is that all functions spanning the space have a zero integral when considering the product of any two modes over the domain. The Neumann modes for the Bay are decent as compared to similar calculations on the unit square and circle.

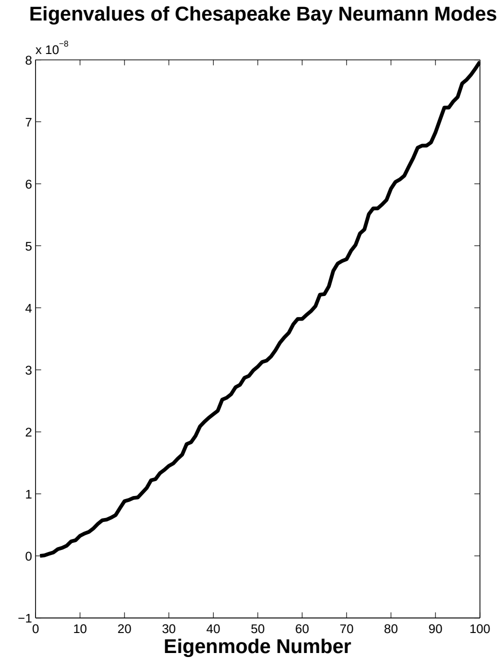


Fig. 7 The progression of eigenvalues for the Neumann modes of the Chesapeake Bay. No fluctuation is observed as the mode number reaches 100, indicating that the feature size of these modes is far from the size of the mesh.

Employing the same techniques for the Neumann modes, agreement could not be reached within an acceptable range (10-20%). While studying the unit square and circle in order to establish baseline performance, the finite difference results typically varied more than FEMLAB, so most likely the error is coming from the finite difference scheme and not FEMLAB. Although not conclusive, the agreement between the two methods indicates stability of the solution set.

Although not shown in this report, the Dirichlet and source terms were calculated for the Chesapeake Bay and can be found in references [8] and [11]. Comparison with the finite difference scheme will require further study to validate either the FEMLAB result or the finite differences result. Given that Neumann conditions are better suited for finite element analysis leads one to trust the FEMLAB results, however, improvements to the finite differences are still possible, which will help validate the FEMLAB solution set. The solutions arising from the source term behaved as expected. For a detailed inquiry into the complete eigenmode set calculated to date for the Chesapeake Bay, consult reference [8]. These results can also be seen at the website listed at USNA [9]. Future work on the velocity vector field for the Chesapeake Bay will include full three dimensional analysis using FEMLAB and Quoddy in conjunction.

Concurrent with this study, the residence time of particulants was analyzed using the Navier-Stokes equations and integrating the velocity fields [10]. By comparing trends in velocity fields from an eigenmode calculation to a Navier-Stokes calculation, one can analyze those parts of the velocity domain that give rise to certain behaviors. This comparison represents the initial steps needed to perform a full Normal Mode Analysis. Plans have begun to further instrument the Bay, leading to the question, how many real-time measurements are required in order to predict important behavior within the Chesapeake. Normal Mode Analysis will be used to help answer this question.

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References

1. Eremeev, V.N., L.M. Ivanov, A.D. Kirwan Jr., ”Reconstruction of Oceanic Flow Characteristics from Quasi-Lagrangian Data - Approach and Mathematical Methods”, J. Geophys. Res., 97, 9733-9742, 1992a.
2. Eremeev, V.N., L.M. Ivanov, A.D. Kirwan Jr., O.V. Melnichenko, S.V. Kochergin, and R.R. Stanichnaya, ”Reconstruction of Oceanic Flow Characteristics from Quasi-Lagrangian Data - Characteristics of the Large-Scale Circulation of the Black Sea”, J. Geophys. Res., 97, 9743-9753, 1992b.

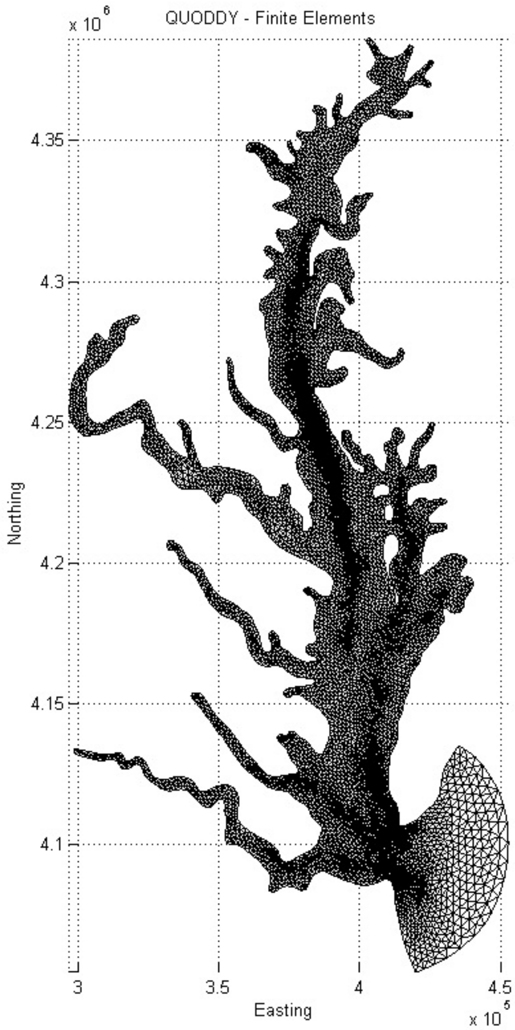

Fig. 18 The mesh from QUODDY [9]. The highest density mesh is in the region of greatest change in the depth.

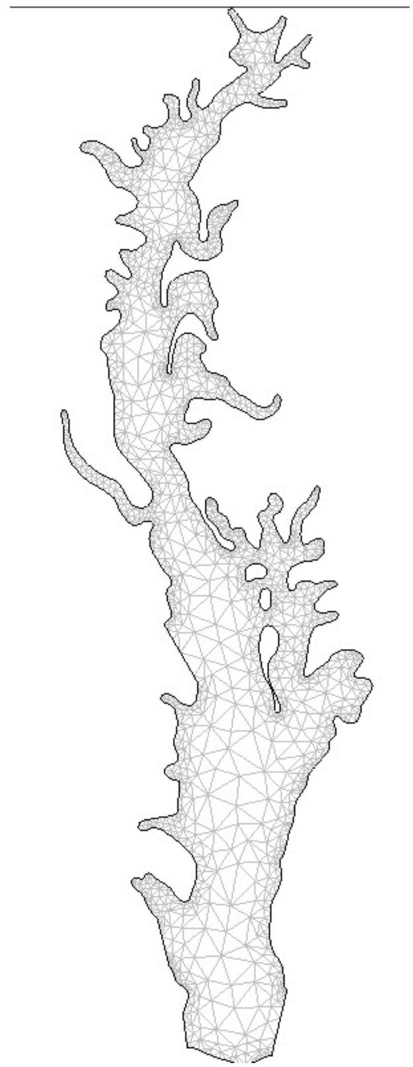

Fig. 19 The mesh from QUODDY's boundary remeshed within COMSOL. The highest density mesh is in the region of greatest change in curvature near the boundary.

cussion of fractals! The boundary for the calculations for this paper were taken from the QUODDY model that the meshes will match. Clearly, the feature set of the boundary has been reduced by averaging the locations of the edges, effectively smoothing the Chesapeake. Even so, the minimal mesh for COMSOL is 3200 elements. By adding in features from the QUODDY mesh, the coarsest mesh will easily be over 5000 elements.

A quick review of oceanographic websites indicates that there are 10 data monitoring stations taking current data in the Chesapeake. This sparse amount of data suggests a data coverage of approximately 0.2%. One possibility for increasing the data coverage is to have many more data taking stations. This is costly and impractical as the Chesapeake is a major waterway for commerce and military use. An alternative approach is to take a truly poor mesh of the Chesapeake Bay. By severely lowering the mesh resolution, the data coverage will increase at the expense of numerical accuracy. By time averaging data, the longer time taken can also overcome poor spatial coverage. Each of these effects suggest an uncertainty principle $\delta(numerics)\ \delta(spatial)\delta(temporal) = constant$. The standard checks for such an analysis will be limited to simple dimensional analysis or 1-D signal applications, as in section 4.

COMSOL is taking projects such as the Chesapeake Bay into new territory as the complexity rises. The bank of tests from simple systems may not easily apply to the richness of the systems capable of being calculated by COMSOL, yet difficult to analyze when combined with real-world data. This is an on-going project. Stay tuned.

Acknowledgements The authors also wish to thank Dr. Tom Gross of the Chesapeake Community Modeling Program and NOAA, Profs. Denny Kirwan and Bruce Lipphardt Of the University of Delaware for their invaluable assistance.

Conclusion

- Possibly found way to use ~10 monitoring stations to extract full Chesapeake Bay flow field.
- Time series has a good chance to work!
- Due to COMSOL and usage of fuller geometric solutions, challenge older precepts based on signal processing.
- If orthonormality is the strongest requirement, create post-processing options to massage results obtained from solvers. Given them tolerances to adjust results.
- Please allow for easier adjustment and creation of meshes!
- Visit us at: http://web.usna.navy.mil/~rmm/

Thanks

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- Prof. Bruce Lipphardt (Univ. Delaware)
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